Petroleum Science 20 (2023) 3441–3449

Contents lists available at ScienceDirect

Petroleum Science

journal homepage: www.keaipublishing.com/en/journals/petroleum-science

Original Paper

Flowback and early-time production modeling of unconventional gas wells using an improved semi-analytical method

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ARTICLE INFO

Article history:
Received 19 August 2022
Received in revised form
6 June 2023
Accepted 9 June 2023
Available online 10 June 2023

Edited by Yan-Hua Sun

Keywords:
Hydraulic fractures
Flowback
Unconventional gas wells

ABSTRACT

Multiple fractured horizontal wells (MFHWs) currently are the only possible means of commercial production from the low and ultra-low permeability unconventional gas reservoirs. In early production time, flowback fluid, which constitutes of hydraulic water and gas flow within fractures, is collected and analyzed. Flowback analysis has been shown to be a useful tool to estimate key properties of the hydraulic fracture such as conductivity and pore volume. Until date, most tools of flowback analysis rely on empirical and approximate methods. This study presents an improved Green-function-based semi-analytical solution for performance analysis of horizontal gas wells during flowback and early production periods. The proposed solution is derived based on coupling the solutions of two domains: a rigorously derived Green's function-based integral solution for single-phase gas flow in matrix, and a finite-difference, multiphase solution for gas–water two-phase flow in the fracture. The validity of proposed semi-analytical solution is verified by finely gridded numerical models built in a commercial simulator for a series of synthetic cases considering a variety of fluid and reservoir property combinations, as well as various different production constraints. Comparisons against available empirical and approximate methods are also provided for these cases.

1. Introduction

In the ultra-low permeability environment found in unconventional plays, multi-transverse hydraulic fractures completed in horizontal wells have constituted the only possible way to produce a commercial quantity of gas. In unconventional reservoirs, fracture characterization is pivotal in production forecasting. One common and straightforward approach is to analyze the early-time data that consists of the fracturing liquid flowback and matrix production following the format of pressure transient analysis (PTA) or rate-transient analysis (RTA) format, from which key fracture parameters such as fracture pore volume, half-length, and conductivity could be determined by straight-line or type-curve methods.

Abassi (2013) quantitatively evaluated the hydraulic fracture properties based on flowback data using a simplified model which assumed single-phase water flow in fracture domain during boundary-dominated (fracture-volume-controlled) period. Clarkson and Williams-Kovacs (2013) and Alkouh et al. (2014) proposed analytical models that considered two-phase water and gas flow in fracture domain under several approximations for the nonlinear phase properties (relative permeability and gas properties) calculations. Ezulike and Dehghanpour (2014) extended the fracture domain-based analytical model and considered single-phase fluid flow from matrix to fracture domain. Zhang and Emami-Meybodi (2020, 2022) further proposed flowback analysis methods applicable to two-phase flow in both fracture and matrix domains. Due to the limitation of the one-dimensional flow model simplification adopted in these studies, one common simplification is the treatment of boundary condition at the matrix-fracture interface, which must be assumed under uniform flux or uniform pressure condition. This approximation could limit the applicability to extreme scenarios where the fracture system exhibits high pressure and saturation gradients.

In recent years, Green's function-based semi-analytical methods have been proposed for the modeling of hydraulically-fractured horizontal wells based on coupling two flow domains (matrix

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https://doi.org/10.1016/j.petsci.2023.06.003
1995-8226/

Nomenclature

B_g	Gas formation volume factor, RB/scf
B_w	Water formation volume factor, RB/STB
c_ϕ	Pore compressibility, psi^{-1}
c_g	Gas compressibility, psi^{-1}
c_{gi}	Gas viscosity at initial condition, cP
c_t	Total compressibility, psi^{-1}
f_a	Analytical adjustment for gas flow, dimensionless
G	Free-space Green's function
h	Reservoir thickness, ft
k_m	Matrix permeability, mD
k_f	Hydraulic fracture permeability, mD
L_f	Hydraulic fracture half length, ft
M_w	Molecular weight, lb/lbmol
m	Normalized real gas pseudopressure, psia
m_D	Dimensionless pseudopressure
n	Number of time steps
p	Pressure, psia
p_{ini}	Initial reservoir pressure, psia
p_{wf}	Bottomhole flowing pressure, psia
$Q_g{}^*$	Source strength in fracture equation, scf/RB/day
$Q_m{}^*$	Source strength in fracture, lb/day/ft^3
$Q_{mD}{}^*$	Dimensionless source strength in matrix
R	Universal constant
S_g	Gas phase saturation
S_w	Water phase saturation
T	Reservoir temperature, °F or °R
t	Time, day
t_D	Dimensionless time
V_b	Bulk volume of fracture element, ft^3
w_f	Fracture width, ft
x	Linear distance under Cartesian system in matrix domain, ft
y	Linear distance under Cartesian system in fracture domain, ft
Z	Gas compressibility factor
λ	Gas viscosity-compressibility ratio, dimensionless
μ_g	Gas phase viscosity, cP
μ_{gi}	Gas phase density at initial condition, cP
μ_w	Water phase viscosity, cP
ρ_g	Gas phase density at reservoir conditions, lb/ft^3
ϕ_m	Matrix porosity
ϕ_f	Fracture porosity

and fracture) (Zhou et al., 2014; Jia et al., 2017; Yang et al., 2018; Zhao et al., 2019) where only discretization along the fractures is required. The fracture discretization enables the rigorous modeling of mass transfer at matrix-fracture interface. The matrix flow is solved using Green's function-based since/sink method which relies on the linearity of the governing diffusivity equation. For nonlinear gas diffusivity equation, available Green's function-based studies all solve the pseudopressure-based gas flow equation while conveniently neglecting the nonlinearity effect caused by retaining pressure-dependent viscosity-compressibility term (Jia et al., 2017; Yang et al., 2018). This retaining nonlinear term, however, has been repeatedly shown to have a significant impact on early-time production behavior of unconventional fractured gas wells (Ibrahim and Wattenbarger, 2006; Nobakht and Clarkson, 2012; Qanbari and Clarkson, 2013; Zhang et al., 2014; Zhang and Ayala, 2018; Nguyen et al., 2020). In the gas PTA and RTA, this pressure-dependent viscosity-compressibility-product of pseudopressure-based gas flow equation has been routinely approximated as time-dependent-only variable so that the pseudotime function applies (Agarwal, 1979). In recent decade, it has been evaluated at the average condition within region-of-influence (ROI) to be applicable for the early-time and flowback analysis of tight and shale gas formations (Qanbari and Clarkson, 2013; Williams-Kovacs and Clarkson, 2016; Clarkson and Qanbari., 2016; Zhang and Emami-Meybodi, 2020).

In our recent work (Zhang and Ayala, 2018), a Green's function-based integral solution is proposed for one-dimensional gas flow problem in matrix and thus applies to gas production from infinite-conductivity fractures. In this solution, the retaining nonlinear viscosity-compressibility in pseudopressure-based gas equation is captured rigorously and straightforwardly via an adjustment factor (f_a). Later in Zhang and Ayala (2019), it has been demonstrated by synthetic and field examples that our f_a-based gas solution can provide more accurate predictions when compared with pseudotime coupled ROI method. In this present study, we further extend this f_a-based semi-analytical solution for 1D linear flow regime to be applicable for the analysis of fracturing liquid flowback and early-time gas production for hydraulically fractured horizontal gas wells. The novel extension is developed based on coupling a series of linear flow in the matrix domain with the finite-difference solution for two-phase gas/water flow in the fracture domain. We consider the proposed solution an improvement as it handles nonlinear, pressure-dependent gas properties and matrix-fracture coupling in a straightforward and accurate manner compared to relevant studies.

2. The improved Green's function-based solution for gas flow in matrix domain

In the analytical or semi-analytical modeling of multi-fractured horizontal wells, one common assumption is that the bi-wing hydraulic fractures are distributed uniformly and with identical properties. Under such conditions, the multi-fractured horizontal well (MFHW) model can be simplified as half of a fracture shown in the Fig. 1. In this study, we assume the outer boundary of matrix domain to be infinite and thus the pressure at outer boundary remains at initial value. This condition applies for MFHWs completed in ultra-low permeability formations before the fracture interference takes place, which is valid for the flowback and early-time production periods (typically within days or weeks). In the matrix domain, the governing diffusivity equation for single-phase gas flow along the x direction shown in Fig. 1 can be expressed as:

$$\frac{\partial}{\partial x}\left(\rho_g \frac{k_m}{\mu_g}\frac{\partial p}{\partial x}\right) = \frac{\partial(\phi_m \rho_g)}{\partial t} + Q_m{}^* \tag{1}$$

In Eq. (1), $Q_m{}^*$ is the source strength in mass per unit volume per unit time, which accounts for the fluid leaving matrix domain to enter hydraulic fracture. Using the normalized gas pseudo-pressure defined by Palacio and Blasingame (1993) and dimensionless group given in Appendix A, Eq. (1) can be expressed in a partially linearized format as:

$$\frac{\partial^2 m_D}{\partial x_D^2} = \frac{1}{\lambda}\frac{\partial m_D}{\partial t_D} + Q_{mD}^* \tag{2}$$

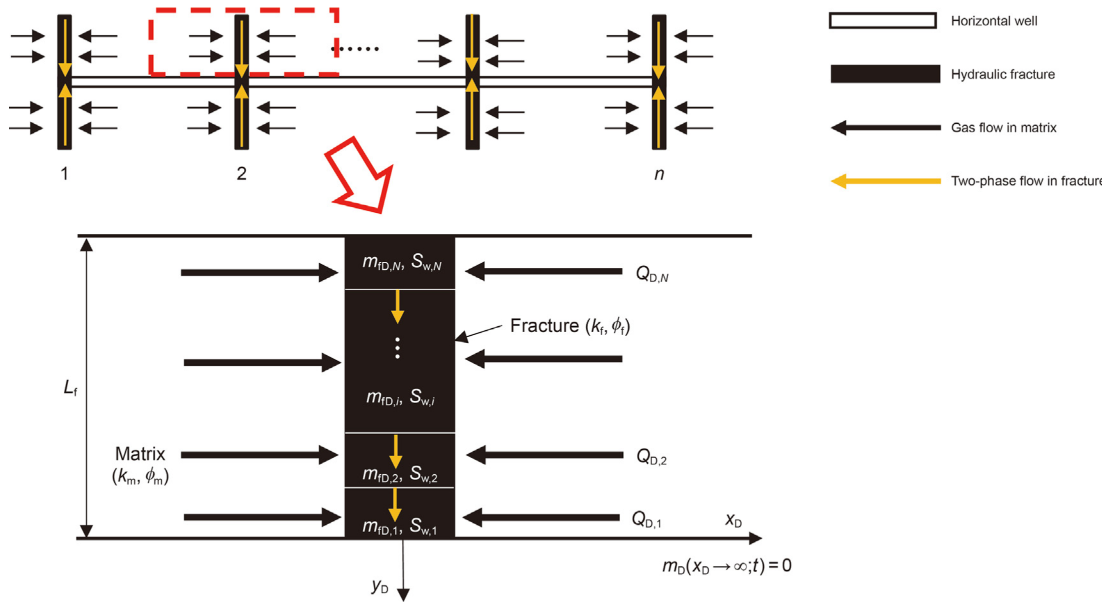

Fig. 1. Schematic of the coupled 1D Cartesian flow regions simulated in this study.

The retaining nonlinearity is the pressure-dependent viscosity-compressibility ratio λ defined as:

$$\lambda = \frac{\mu_{gi} c_{ti}}{\mu_g(p) c_t(p)} \tag{3}$$

where μ_g is the gas viscosity; c_t is the total compressibility and equals the summation of gas compressibility (c_g) and pore compressibility (c_ϕ). In the state-of-the-art semi-analytical solution for the flowback analysis of fractured gas wells (Jia et al., 2017; Yang et al., 2018), λ is always assumed to be constant and equal to unity. Under such assumption, the solution of Eq. (2) can be straightforwardly found by the classical Duhamel's principle:

$$m^*_{fD}(0, t_D) = \int_0^{t_D} \frac{Q_{mD}(\tau)}{\sqrt{4\pi(t_D - \tau)}} d\tau \tag{4}$$

However, for the early time production of fractured gas wells, such simplification of neglecting the nonlinear effect of λ has been repeatedly proved to lead to erroneous predictions for the recovery performance (Ibrahim and Wattenbarger, 2006; Qanbari and Clarkson, 2013). In Zhang and Ayala (2018) and Nguyen et al. (2020), an adjustment factor (f_a) has been proposed to capture the nonlinear, pressure-dependent λ in a rigorous and direct manner:

$$f_a(t_D) = \int_0^{t_D} d\tau \int_{-\infty}^{+\infty} \left[\left(\frac{1}{\lambda(\xi, \tau)} - 1 \right) \frac{\partial m_D(\xi, \tau)}{\partial t_D} G \right] d\xi \tag{5}$$

G is the free-space Green's function defined as:

$$G(t_D; \xi, \tau) = \frac{1}{\sqrt{4\pi(t_D - \tau)}} \exp\left[-\frac{\xi^2}{4(t_D - \tau)} \right] \tag{6}$$

Zhang and Ayala (2018) and Nguyen et al. (2020) have shown that, for gas horizontal wells with infinite-and finite-conductivity fractures, Eq. (4) fails to provide the reliable predictions during early-time production periods, and predictions are significantly improved by using the adjustment factor f_a given by Eq. (7). It is the interest of this work to further extend this improved Green-function-based semi-analytical solution to the flowback and early-time production periods where two-phase (water/gas) fracture flow is considered.

For the discretized system shown in Fig. 1, the response at the location of i-th fracture element at n-th time step is given as:

$$m^{(n)}_{fD,i} = m^{*(n)}_{fD,i} + f^{(n)}_{a,i}; \quad i = 1, 2, 3, \ldots, N \tag{7}$$

with

$$m^{*(n)}_{fD,i} = \int_0^{t_D} \frac{Q_{mD,i}(\tau)}{\sqrt{4\pi(t_D - \tau)}} d\tau \cong \sum_{j=1}^{n} \frac{Q^{(j)}_{mD,i}}{\sqrt{\pi}} \sqrt{(t_{D,n} - t_{D,j})} \tag{8}$$

Eq. (7) provides a system of N equations in which only unknowns at fractures, namely $m_{fD,i}$ and $Q_{mD,i}$ of each element, need to be solved. This $2N$ unknowns are later be solved by coupling Eq. (7) with fracture flow equations.

3. The finite difference solution for two-phase flow in fracture domain

Under the assumption of gravity segregated relative permeability, k_{rg} and k_{rw}, are equal to S_g and S_w, respectively, the governing diffusivity equations of one-dimensional gas–water flow within a fracture along the y axis shown in Fig. 1 are written as:

$$\frac{\partial}{\partial y}\left(\frac{k_f S_g}{B_g(p)\mu_g(p)} \frac{\partial p}{\partial y} \right) + Q^*_g = \frac{\partial}{\partial t}\left(\frac{\phi_f S_g}{B_g} \right) \tag{9}$$

$$\frac{\partial}{\partial y}\left(\frac{k_f S_w}{B_w \mu_w} \frac{\partial p}{\partial y} \right) = \frac{\partial}{\partial t}\left(\frac{\phi_f S_w}{B_w} \right) \tag{10}$$

For gas equation, the term Q_g denotes the gas influx from matrix to fracture, which is embedded in the dimensionless source strength Q_{mD} given in Appendix A. The governing equations for i-th element at time step n in discrete form would be:

$$\frac{k_\mathrm{f}\left(S_{\mathrm{g},i}^n\right)w_\mathrm{f}h}{B_\mathrm{g}\left(p_{i-1/2}^n\right)\mu_\mathrm{g}\left(p_{i-1/2}^n\right)}\frac{p_{i-1}^n-p_i^n}{\Delta y_{i-1/2}}+\frac{k_\mathrm{f}\left(S_{\mathrm{g},i+1}^n\right)w_\mathrm{f}h}{B_\mathrm{g}\left(p_{i+1/2}^n\right)\mu_\mathrm{g}\left(p_{i+1/2}^n\right)}\frac{p_{i+1}^n-p_i^n}{\Delta y_{i+1/2}}+Q_{\mathrm{g},i}=\frac{\phi_\mathrm{f}V_{\mathrm{b},i}}{\Delta t}\left(\frac{S_{\mathrm{g},i}^n}{B_\mathrm{g}\left(p_i^n\right)}-\frac{S_{\mathrm{g},i}^{n-1}}{B_\mathrm{g}\left(p_i^{n-1}\right)}\right) \tag{11}$$

$$\frac{k_\mathrm{f}\left(S_{\mathrm{w},i}^n\right)w_\mathrm{f}h}{B_\mathrm{w}\mu_\mathrm{w}}\frac{p_{i-1}^n-p_i^n}{\Delta y_{i-1/2}}+\frac{k_\mathrm{f}\left(S_{\mathrm{w},i+1}^n\right)w_\mathrm{f}h}{B_\mathrm{w}\mu_\mathrm{w}}\frac{p_{i+1}^n-p_i^n}{\Delta y_{i+1/2}}=\frac{\phi_\mathrm{f}V_{\mathrm{b},i}}{\Delta t}\left(\frac{S_{\mathrm{w},i}^n-S_{\mathrm{w},i}^{n-1}}{B_\mathrm{w}}\right) \tag{12}$$

where $V_{\mathrm{b},i}$ is the bulk volume of i-th element. For each element, there are three primary unknowns: pressure, matrix production, and saturation. Implicit pressure explicit saturation method, or IMPES, is implemented to explicitly calculate saturation at iteration level $a+1$ and time step n as a function of pressure and saturation from previous iteration level a and previous time step $n-1$, using water governing equation:

$$S_{\mathrm{w},i}^{n^{(a+1)}}=\left(\frac{k_\mathrm{f}\left(S_{\mathrm{w},i}^{n^{(a)}}\right)w_\mathrm{f}h}{B_\mathrm{w}\mu_\mathrm{w}}\frac{p_{i-1}^{n^{(a+1)}}-p_i^{n^{(a+1)}}}{\Delta y_{i-1/2}}+\frac{k_\mathrm{f}\left(S_{\mathrm{w},i+1}^{n^{(a)}}\right)w_\mathrm{f}h}{B_\mathrm{w}\mu_\mathrm{w}}\frac{p_{i+1}^{n^{(a+1)}}-p_i^{n^{(a+1)}}}{\Delta y_{i+1/2}}\right)\frac{\Delta t}{\phi_\mathrm{f}V_{\mathrm{b},i}}B_\mathrm{w}+S_{\mathrm{w},i}^{n-1} \tag{13}$$

A total of $2N$ fracture flow equations (Eqs. (11) and (12)), with pressure and matrix influx for each element as unknown, are coupled with N matrix flow equations (Eq. (7)) to obtain the simultaneous solution of saturation, dimensionless pseudopressure and source strength of N fracture elements ($S_{\mathrm{w}i}$, $m_{\mathrm{fD},i}$, and $Q_{\mathrm{mD},i}$ of each element). Conversion between dimensional and dimensionless properties is proposed by Zhang and Ayala (2018) and provided in Appendix A. A simple Newton–Raphson procedure is deployed to obtain the final solutions. Appendix B provides a detailed flow chart for this solution method.

The coupled solution assumes only 1D flow from matrix to fracture and neglects all fluid interaction inside matrix. This is reasonable assumption when fracture permeability is many orders of magnitude higher than matrix permeability, which is a common condition in unconventional gas reservoir. The extension to 2D gas flow in matrix domain, however, requires further modifications of adjustment factor f_a (Eq. (5)) to rigorously capture the nonlinearity effect in a 2D flow geometry, and will be further discussed in our future work.

4. Application to multi-fractured horizontal gas wells: Comparison with numerical simulations

In this section, comparison is first made between the proposed semi-analytical solution and common industry practice of using harmonic decline curve for flowback analysis. Next, validation of the proposed coupled solution is conducted by matching against finely gridded numerical simulation model built in Computer Modeling Group (CMG). We also provide the comparisons with the most state-of-the-art Green-function-based semi-analytical solution for flowback and early-time production analysis, in which the pressure-dependency of viscosity-compressibility is neglected in matrix gas flow (Jia et al., 2017; Yang et al., 2018). Two synthetic fluids are studied (Table 1), including one high-pressure high-temperature (HPHT) reservoir conditions, to study the nonlinear effect due to pressure-dependent gas properties. Both constant and variable pressure/rate scenarios are presented. In the semi-analytical solutions and numerical simulations, gas properties are calculated using Dranchuk and Abou-Kassem (1975) for Z-factor, Lee et al. (1966) for gas viscosity, Abou-Kassem et al. (1990) for gas isothermal compressibility, and Sutton (1985) for pseudo-critical property calculations.

4.1. Benchmark case: Comparison with common industry practice

Fetkovich (1971) presented the harmonic decline model based on single-phase water volumetric depletion to analyze and forecast performance of water drive reservoir. The harmonic decline model is presented with two variables: D_i as initial decline coefficient and q_i as initial decline rate.

$$q=\frac{q_\mathrm{i}}{1+D_\mathrm{i}t} \tag{14}$$

In recent years, Eq. (14) has been commonly applied in the industry practice of flowback analysis as the empirical model to analyze flowback data and estimate pore volume of fracture, as remarked in Fu et al. (2019). The terms D_i and q_i are calculated as:

$$D_\mathrm{i}=\frac{0.00633k_\mathrm{f}}{\phi_\mathrm{f}\mu_\mathrm{w}c_\mathrm{t}L_\mathrm{f}^2} \tag{15}$$

$$q_\mathrm{i}=\frac{kh\left(p_\mathrm{ini}-p_\mathrm{wf}\right)}{141.2\mu_\mathrm{w}B_\mathrm{w}\frac{L_\mathrm{f}}{w_\mathrm{f}}} \tag{16}$$

Taking Case 1 as example, we provide the comparisons for both constant and variable BHP scenarios. For the variable pressure scenario, the schedule is 3000 psia until 150 days, followed by 1000 psia until 1000 days. Comparisons of water rates from three methods: proposed solution, harmonic decline model, and numerical model, are displayed in Fig. 2. Solid blue lines represent results generated using CMG, and proposed solution is shown as red dot line. Green dash lines are estimation obtained from harmonic decline method. Excellent matches between numerical model and proposed solution for water rate can be observed in both early and late time. On the other hand, the rate calculated by harmonic decline model shows significant discrepancy. This is expected, as the simple decline model is based on single-phase water depletion, which does not consider contribution from *in-situ* gas in

Table 1
Input properties for case studies.

Parameter	Case 1	Case 2 (HPHT)
Initial pressure p_ini, psia	5000	16,200
Temperature T, °F	200	300
Specific gravity	0.95	0.633
Matrix permeability k_m, mD	9×10^{-6}	
Matrix pore compressibility c_ϕ, psi^{-1}	1×10^{-6}	
Matrix porosity ϕ_m	0.0661	
Thickness h, ft	200	
Hydraulic fracture half length L_f, ft	300	
Hydraulic fracture permeability k_f, mD	50	
Hydraulic fracture porosity ϕ_f	0.9	
Hydraulic fracture width w_f, ft	0.01	
Initial water saturation in fracture $S_\mathrm{w,ini}$	0.8	

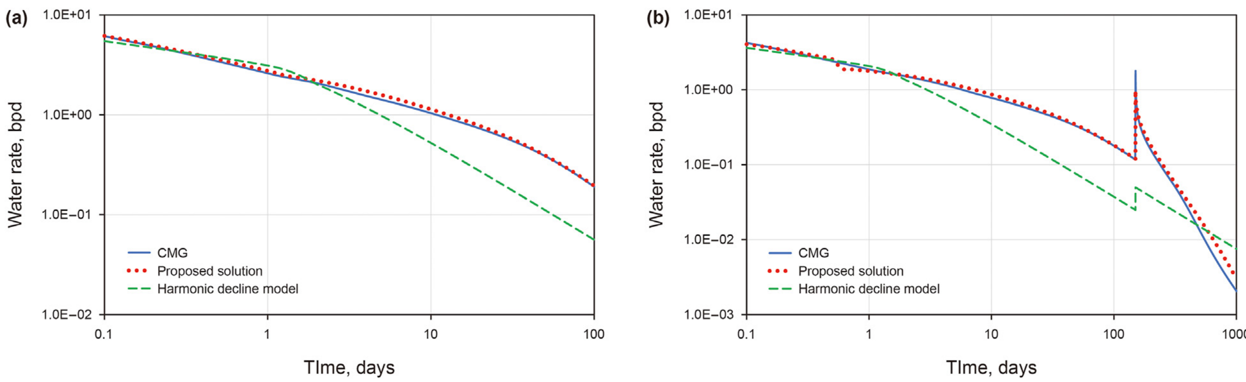

Fig. 2. Water rate match for constant BHP (**a**) and variable BHP (**b**).

fracture and gas flow from matrix. Hence, the harmonic decline could be considered an approximate model during very early periods of flowback but will fail terribly in capturing the production behavior during late-time flowback and early-time production of fractured gas wells.

4.2. Application #1: Constant bottomhole pressure constraint

Application #1 studies the flowback and early production behavior of fractured gas wells under constant bottomhole pressure. Figs. 3 and 4 compare the gas rate versus time for normal reservoir condition, obtained from the proposed coupled solution, finely gridded numerical model using CMG, and state-of-the-art semi-analytical solution proposed by Jia et al. (2017) and Yang et al. (2018), in which the viscosity-compressibility product λ in matrix is assumed to be constant and equal to unity. Solid blue lines represent results of CMG, while proposed solution and semi-analytical solution with $\lambda = 1$ is shown as red dot line and green dash line, respectively. The pressure drop is 3000 psia, from the initial pressure of 5000 psia. Overlapping results can be observed for proposed solution and CMG. This indicates that the solution can accurately predict the fracturing water flowback and gas early-production behavior of fractured gas wells by coupling a direct integral solution for matrix gas flow with a finite-difference, multiphase model for fracture gas–water flow. On the other hand, the solution with approximation of $\lambda = 1$ shows noticeable deviation, especially toward late time. This is because this solution fails to capture the retaining nonlinear, pressure-dependent gas viscosity-compressibility term in the matrix gas flow equation. The impact of this practice is more significant at higher pressure drawdown when λ deviates significant from unity, which will be demonstrated in later application.

In Fig. 3, water rates are only provided until 100 days, and gas rates are shown until 1000 days. After this time, water rate is extremely low and longer plotted time can obscure the rate in early time (< 50 days). Unlike gas rate comparisons, almost all lines overlap in both early and late time in water rates. As expected, the proposed solution, which rigorously captures both fracture and matrix response, matches perfectly with CMG numerical model. The semi-analytical solution with $\lambda = 1$, while unable to match with gas rate shows good agreement in water rate with the other two solutions. The negligence of viscosity-compressibility product only happens in matrix gas flow. Inside fracture, the nonlinearity of gas properties is still considered in the finite-difference multiphase solution.

Gas and water responses for HPHT conditions are graphed in Fig. 4, with constant pressure drawdown of 8200 psia. Similar to normal reservoir condition, for gas rate, excellent match is again observed between the proposed solution and numerical model from CMG, while the semi-analytical solution with $\lambda = 1$ shows discrepancy at late time. The water rates of all solutions are well matched.

4.3. Application #2: Variable bottomhole pressure constraint

Figs. 5 and 6 compare the well gas and water rates of Case 1 predicted by the proposed solution and CMG's numerically simulated results. The legends are the same as that of graphs in constant bottomhole pressure case, with blue solid lines represent CMG, red dot lines are for the proposed solution, and solution with approximation of $\lambda = 1$ is denoted as green dash lines. The bottomhole pressure schedule is 3000 psia until 150 days, followed by 1000 psia until 1000 days for Case 1. The impact of neglecting the pressure-dependent properties in matrix gas flow is clearly

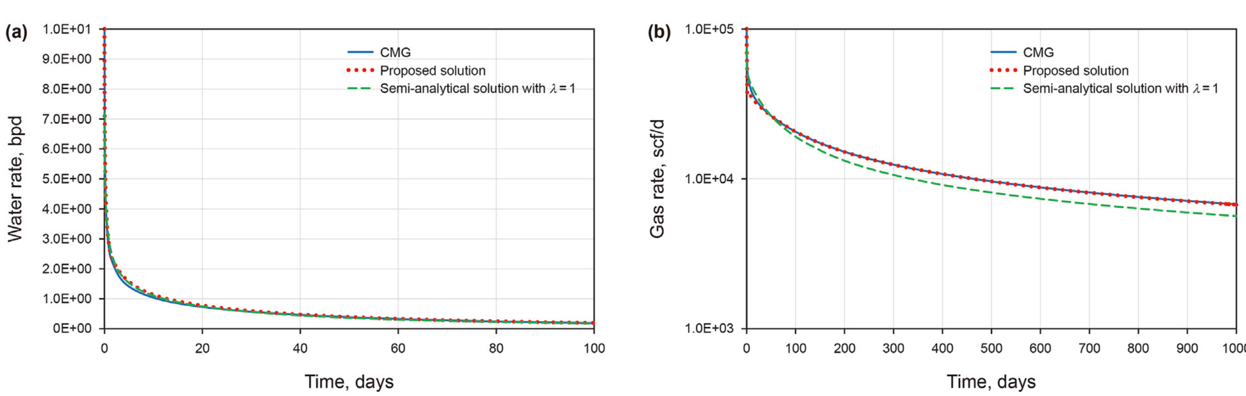

Fig. 3. Water rate (**a**) and gas rate (**b**) for constant BHP (Case 1).

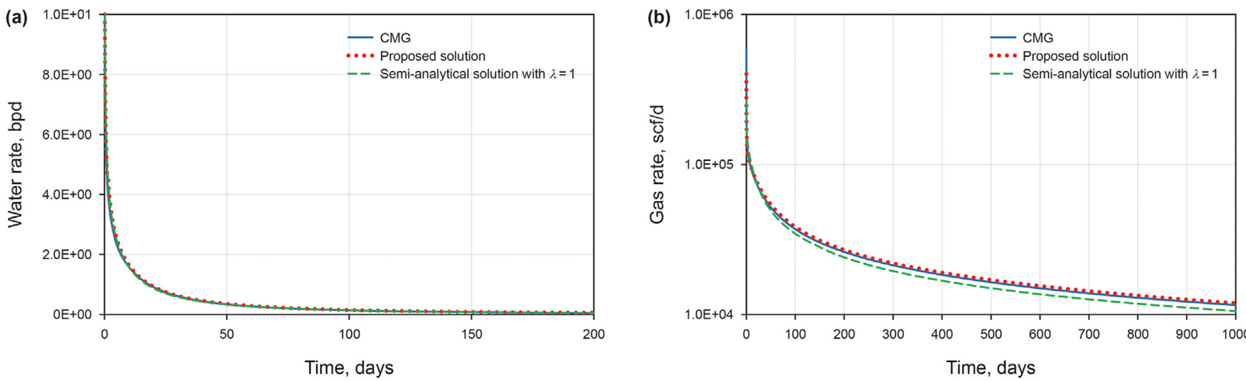

Fig. 4. Water rate (**a**) and gas rate (**b**) for constant BHP (Case 2).

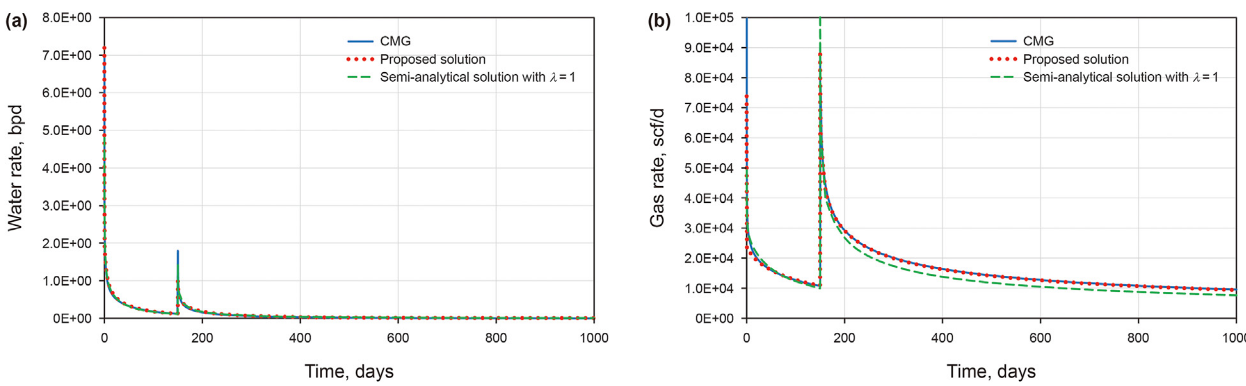

Fig. 5. Water rate (**a**) and gas rate (**b**) under variable BHP (Case 1).

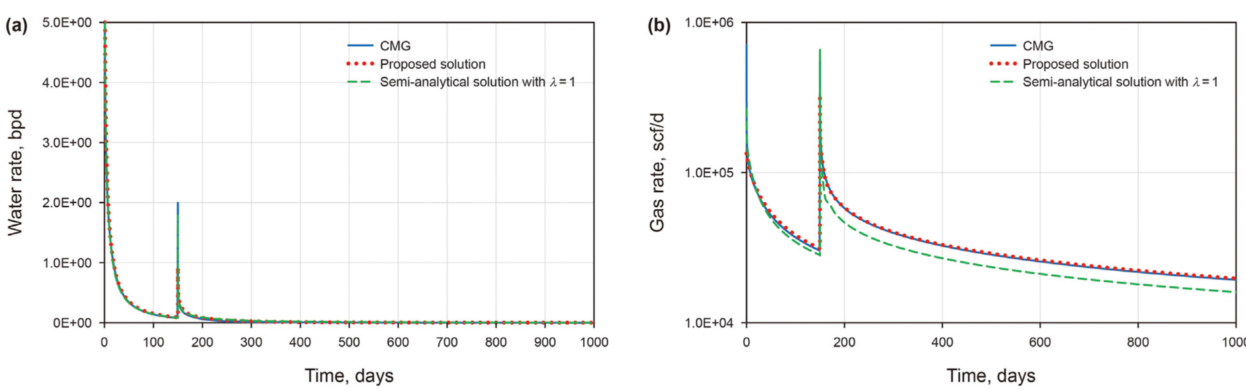

Fig. 6. Water rate (**a**) and gas rate (**b**) under variable BHP (Case 2).

observed after BHP changes at 150 days, as the $\lambda = 1$ approximate solution fails to match the proposed solution and simulated data from CMG. Similar to previous application example, the impact of approximating $\lambda = 1$ on water rate is rather negligible, as the two-phase water/gas solution for fracture flow captures all nonlinearity terms with finite-difference scheme.

Similar responses of both gas and water rates for HPHT reservoir condition can be observed in Fig. 6 for Case 2. Bottomhole pressure is kept at 8000 psia before 150 days then reduced to 4000 psia. The effect of $\lambda = 1$ on production response has not been significant, with discrepancy only noticeable in late time. This can be explained as fracture pressure drops to specified bottomhole pressure quickly due to high fracture conductivity. To further demonstrate the ability of our solution to capture reservoir behavior in all possible scenarios, the next application will compare bottomhole pressure response under the constraint of gas rate specification.

4.4. *Application #3: Constant gas rate constraint*

Under constant gas rate production constraint, bottom-hole pressure responses and water rates for Case 1 are shown in Figs. 7 and 8. The specified gas rate for this application is 28 Mscfd. Results given by proposed solution and simulation results of CMG are excellently matched. On the other hand, the semi-analytical solution with approximation of $\lambda = 1$ starts to deviate from accurate predictions after only 200 days and continues to underestimate

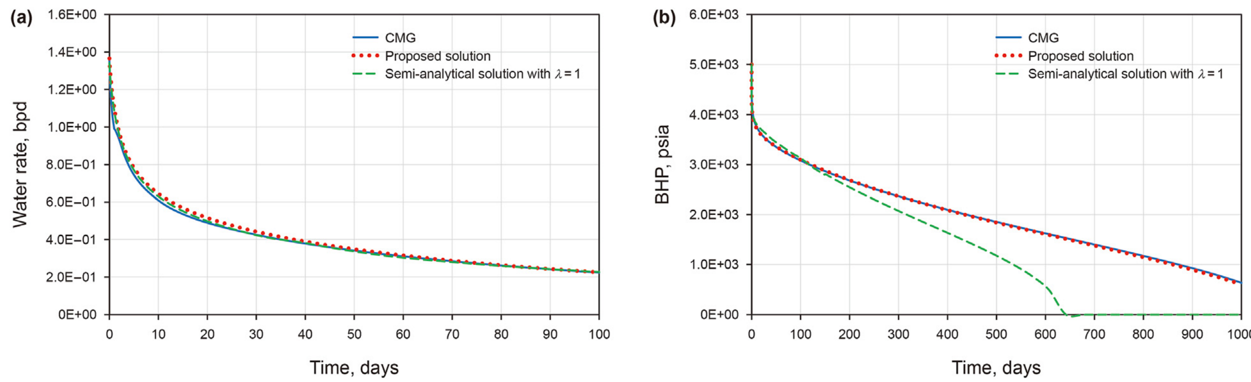

Fig. 7. Water rate (**a**) and BHP (**b**) under constant gas rate condition (Case 1).

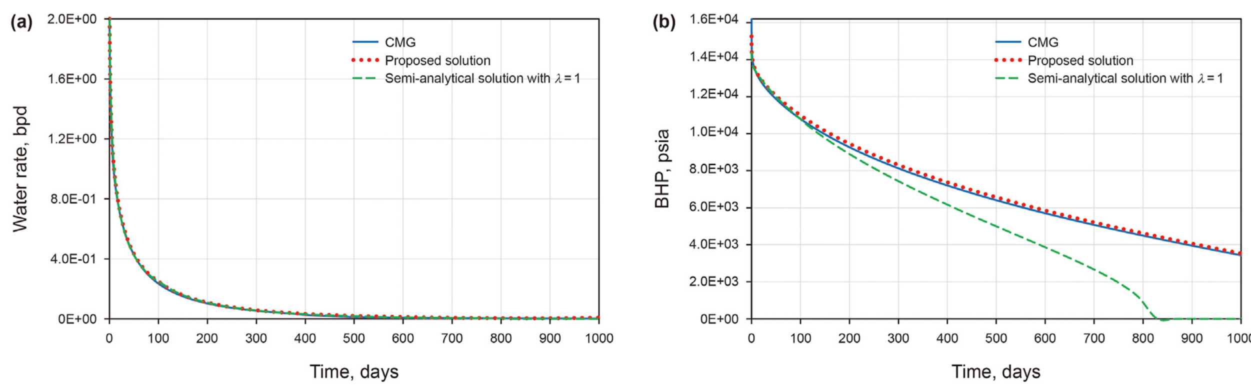

Fig. 8. Water rate (**a**) and BHP (**b**) under constant gas rate condition (Case 2).

the pressure responses until reaching unrealistic values (< 14.7 psia). This is because neglecting the pressure-dependency of viscosity-compressibility product λ in matrix will reduce capability of reservoir to maintain a high gas rate, resulting in higher pressure drop. This behavior has been reported by Zhang and Ayala (2018) for production from infinite conductivity hydraulic fracture and by Nguyen et al. (2020) for production from finite conductivity hydraulic fracture and now further confirmed by this work which extends to flowback periods. By considering nonlinearity of gas properties in matrix, the proposed solution can match with numerical simulation results in CMG. The water rates, which are fairly independent of matrix gas flow influence, closely match for all three solutions.

For Case 2 (HPHT reservoir condition), similar observations can be made as that of Case 1. Under the specified gas rate of 60 Mscfd, the bottomhole pressure predicted by the approximate semi-analytical solution with $\lambda = 1$ drops dramatically after a short time, while results predicted by proposed solution and CMG are well matched. The water rate predicted by all solutions are very close, as explained previously.

4.5. Application #4: Variable gas rate constraint

This section presents the results of water rate and BHP response under variable gas rate specifications. The gas production rate for Case 1 is kept at 28 Mscfd till 150 days, followed by 14 Mscfd. Fig. 9 displays the BHP response generated by three methods: the proposed coupled solution, numerical model from CMG and the semi-analytical solution with approximation of $\lambda = 1$. It is demonstrated that the proposed solution can provide the most accurate predictions by matching the simulated results from CMG. While the BHP predicted by the approximation solution with $\lambda = 1$ can keep up with the other two solutions until 150 days, it drops noticeably after the rate change due to the assumption that viscosity-compressibility remaining at initial condition in matrix gas flow equation. The water rate of this approximate solution is well matched with those from the proposed solution and CMG.

Fig. 10 shows the results of Case 2. The gas rate is kept at 80 Mscfd initially and later drops to 30 Mscfd at 150 days. Similar behavior of bottomhole pressure and water rate predictions are observed as that in Case 1, with the bottomhole pressure predicted by semi-analytical solution with $\lambda = 1$ falls much faster than proposed solution and CMG after the rate change at 150 days, while the water rate of three solutions are closely matched.

5. Conclusions

This work proposes an improved Green's function-based semi-analytical solution applicable for the flowback and early transient behavior analysis of multi-fractured horizontal gas wells. The proposed solution is developed by coupling two solutions for two neighboring domains: matrix and fracture. The governing equation for single-phase matrix gas flow is solved using a direct integral Green's function-based solution, which rigorously captures the retaining nonlinearity due to viscosity-compressibility product in the pseudopressure-based gas diffusivity equation. The gas–water two-phase flow governing equations in fracture are solved using finite-difference method and IMPES algorithm. The two solutions are then coupled to obtain a simultaneous solution for pressure, saturation, and gas influx from matrix to fracture. Validations are conducted using finely gridded numerical simulation models from commercial numerical simulator CMG. A series of case studies are

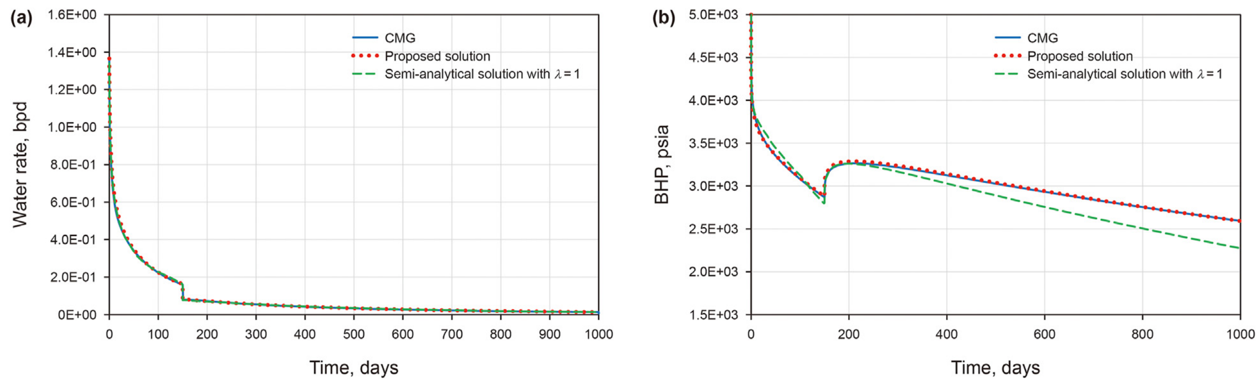

Fig. 9. Water rate (**a**) and BHP (**b**) under variable gas rate condition (Case 1).

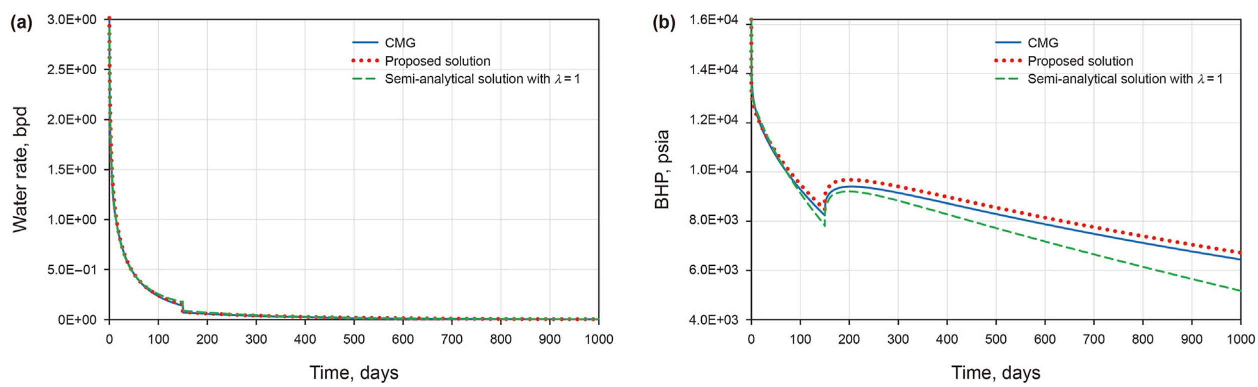

Fig. 10. Water rate (**a**) and BHP (**b**) under variable gas rate condition (Case 2).

presented, including constant and variable p/q production conditions for regular and HPHT gas reservoirs. Results show that for all the cases tested, excellent matches are found between the proposed solution and numerical simulation results. We further compare the proposed solution with the state-of-the-art methods which approximately assume the remaining gas viscosity-compressibility term constant and equal to initial value. Results show that neglecting this nonlinear effect by wrongfully assuming it is remaining at initial value will lead to overestimation of pressure drop under rate-specified condition, and underestimation of gas rates under pressure-specified condition. This important observation corroborates the superiority of the proposed solution being an accurate and reliable model for the flowback and early transient behavior analysis of multi-fractured horizontal gas wells under a variety of production conditions. This present work focuses on the forward-looking examples and uses synthetic data obtained from numerical simulation models for validation purpose. Further extension of the proposed method to inverse analysis of field data in the format of straight-line or type-curve is part of the ongoing work currently undertaken by our research group.

The novelty of this study is the extension of the f_a-based gas solution, which is previously proposed by our research group for 1D Cartesian gas flow in matrix only, to be applicable to flowback analysis of unconventional fractured gas wells. The nonlinearities of gas flow in matrix are limited to the pressure-dependent gas properties. Future work includes the further considerations of nonlinear effects due to other possible sources like the transport and storage mechanisms in extremely tight, nanoporous shale gas reservoirs such as diffusion and sorption.

Declaration of competing interest

Authors declare no conflict of interest.

Acknowledgements

The authors acknowledge the support from National Natural Science Foundation of China (No. 52174042) and China University of Petroleum Beijing (No. 2462021YXZZ011, No. PRP/indep-4-2113) for the completion of this study.

Appendix A. Dimensionless variable group

Table A1 provides the definitions of the dimensionless variables used in this manuscript.

Table A1
Dimensionless variable group

t_D	x_D	y_D	m_D	Q_{mD}
$\frac{k_m t}{\phi_m \mu_{gi} c_{ti} L_f^2}$	$\frac{x}{L_f}$	$\frac{y}{L_f}$	$\frac{m_i - m}{m_i}$	$\frac{Q_m^* RT \mu_{gi} Z_i L_f}{M_W p_i k_m m_i}$

Appendix B. Flow chart of proposed semi-analytical solution

Fig. B1 showcases the detailed flow chart of employing proposed semi-analytical method to solve coupled matrix and fracture equations simultaneously.

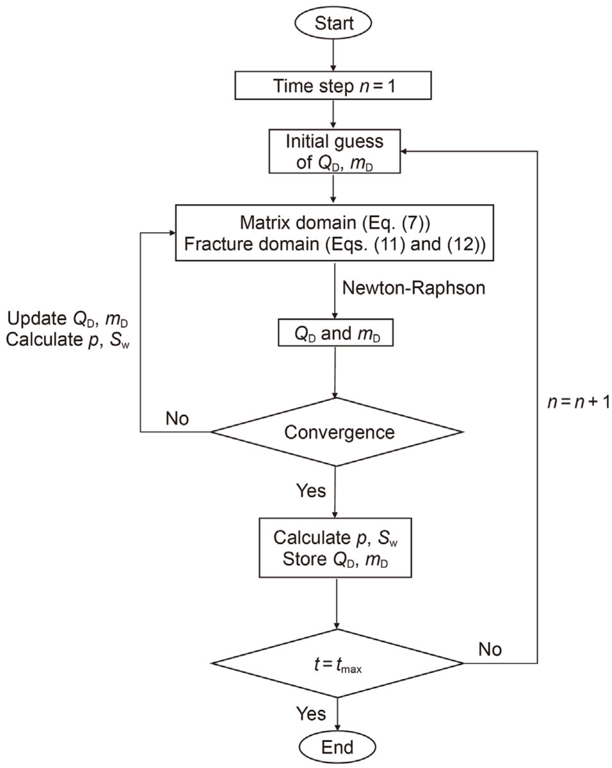

Fig. B1. Detailed flow chart of proposed semi-analytical solution

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