Novel empirical correlations for estimation of bubble point pressure, saturated viscosity and gas solubility of crude oils

Ehsan Khamehchi[1], Fariborz Rashidi[1]*, Hanieh Rasouli[1] and Amir Ebrahimian[2]

[1] Faculty of Chemical Engineering, Amirkabir University of Technology, Tehran, Iran
[2] Faculty of Petroleum Engineering, Amirkabir University of Technology, Tehran, Iran

Abstract: Knowledge of petroleum fluid properties is crucial for the study of reservoirs and their development. Estimation of reserves in an oil reservoir or determination of its performance and economics requires a good knowledge of the fluid physical properties. Bubble point pressure, gas solubility and viscosity of oils are the most important parameters in use for petroleum and chemical engineers. In this study a simple-to-use, straight-forward mathematical model was correlated on a set of 94 crude oil data. Three correlations were achieved based on an exponential regression, which were different from conventional empirical correlations, and were evaluated against 12 laboratory data other than those used for the regression. It is concluded that the new exponential equation is of higher precision and accuracy than the conventional correlations and is a more convenient mathematical formulation.

Key words: Bubble point pressure, saturated viscosity, gas solubility, empirical equation, exponential multiple regression

1 Introduction

Determining the fluid properties by utilizing the PVT analysis is considered to be essential in the study of petroleum reservoirs, the design of production equipment and the estimation of the recovery efficiency of a reservoir. In the absence of these data for one reason or another, correlations are used to estimate the physical properties of a petroleum fluid.

It is worth mentioning that during the study of pressure losses that occur in a multiphase flow situation, there is the need to know the physical properties of a reservoir fluid at different temperatures, which are always below the temperature of the reservoir.

These properties are also needed for the design of surface operation equipment or the study of reservoir flow performance. Therefore correlations for estimating the values of these properties must be used.

2 Petroleum fluid properties

Reservoir engineers need to predict PVT properties of petroleum fluids at given conditions of temperature and pressure. Though this can be determined through PVT analysis of fluid sample tests or can be calculated by using equation-of-state based on computer codes if the fluid composition is known, this information is often unavailable particularly at the early stage of field development or needs to be verified, supported and supplemented during the course of field development. It is the task of empirical correlations then to estimate the petroleum fluid properties as a function of the reservoir's readily available characteristics (Ahmed, 1990).

3 Methods for predicting bubble point pressure

In a given hydrocarbon system the highest pressure at which the first gas bubble forms is called the bubble point or the 'saturation' pressure of that system. This can be experimentally obtained by constant composition expansion tests.

Standing (1981) proposed the following correlation:

$$P_b = 18.2\left[\left(R_s\Big/\gamma_g\right)^{0.83}(10)^a - 1.4\right] \quad (1)$$

$$a = 0.00091(T_{(^\circ\mathrm{R})} - 460) - 0.0125(\mathrm{API})$$

Glaso (1981) also proposed the following correlation:

$$Log(P_b) = 1.7669 + 1.7447 \log(P_b^*) - 0.30218\left[\log(P_b^*)\right]^2 \quad (2)$$

where P_b^* is given by the following equation:

*Corresponding author. email: rashidi@aut.ac.ir

$$P_b^* = \left(R_s \big/ \gamma_g \right)^a (T)^b (API)^c \tag{3}$$

a=0.816, b=0.172, c=−0.989

Furthermore, Marhoun (1988) proposed the following relationship based on Middle Eastern crude oils:

$$P_b = aR_s^b\gamma_g^c\gamma_o^d T^e \tag{4}$$

a=5.38088E-3, b=0.715082, c=−1.87784
d=3.1437, e=1.32657

4 Methods for predicting oil viscosity

Viscosity of reservoir fluids is one of the factors that determine the degree of the fluids being mobilized within reservoir rock. Accurate prediction of viscosity should be made at a given temperature and pressure so that proper enhanced or natural oil recovery scenarios could be planned and right mechanisms could be utilized after satisfying predictions are proved through simulation.

Regardless of the specific lab experiments that are usually requested for a reservoir, one can find well-known correlations for this purpose in the literature.

For dead oil, Beal (1946) proposed a graphical correlation in 1946 that was later transformed into a mathematical form by Standing in 1981 (see Eq. 5):

$$\mu_{od} = \left(0.32 + \frac{1.8\times10^7}{API^{0.453}} \right)\left(\frac{360}{T-260} \right)^a \tag{5}$$

In which $a = 10^{(0.43+0.83/API)}$

Beggs and Robinson (1975) also proposed the following correlation (Eq. 6) for dead oil, while later Sutton and Farshad (1984) reported 114.7% of error for this correlation when tested against 93 crude samples from the literature.

$$\mu_{od} = 10^x - 1 \tag{6}$$

$$X = Y(T-460)^{-1.163}$$

$$Y = 10^z$$

$$Z = 3.0324 - 0.02023\,API$$

The correlations proposed for calculation of viscosity of saturated oils are commonly based on the viscosity of oil in dead state. Chew and Connally (1959) presented a correlation that was transformed into a mathematical equation later by Standing in 1981 (Eq. 7).

$$\mu_{ob} = 10^a (\mu_{od})^b \tag{7}$$

$$a = R_s\left[2.2\left(10^{-7}\right)R_s - 7.4\left(10^{-4}\right) \right]$$

$$b = \frac{0.68}{10^c} + \frac{0.25}{10^d} + \frac{0.062}{10^e}$$

$$c = 8.62\left(10^{-5}\right)R_s$$

$$d = 1.1\left(10^{-3}\right)R_s$$

$$e = 3.74\left(10^{-3}\right)R_s$$

Beggs and Robinson (1975) suggested the following correlation

$$\mu_{ob} = a\left(\mu_{od}\right)^b \tag{8}$$

$$a = 10.715\left(R_s + 100\right)^{-0.515}$$

$$b = 5.44\left(R_s + 150\right)^{-0.338}$$

5 Methods for predicting gas solubility

Gas solubility (R_s) is a measure of the capability of oil to dissolve gas. It is in nature the result of complicated molecular interaction between all the individual components existing in oil and in associated gas. But, it is usually experimentally determined in the laboratory. For this oil property also there are empirical correlations presented in the literature.

Standing (1981) suggested the following equation:

$$R_s = \gamma_g\left(\left(\frac{P}{18.2} + 1.4 \right)10^x\right)^{1.2048}$$

$$x = 0.0125\,API - 0.00091(T-460)$$

Petrosky and Farshad (1993) suggested the following equation based on crude oil data from the Gulf of Mexico:

$$R_s = \left[\left(\frac{P}{112.727} + 12.340 \right)\gamma^{0.8439}10^x\right]^{1.73184}$$

$$x = 7.916(10^{-4})API^{1.5410} - 4.561(10^{-5})(T-460)^{1.3911}$$

6 Multiple regression analysis

Multiple regression analysis attempts to explain the relationships between the independent variables and a dependent variable. When there are p independent variables $X_1, X_2, \cdots, X_p$, the multiple exponential regression equation is in the general form as follows:

$$Y_i = \beta_0 X_1^{\beta1} X_2^{\beta2}\cdots X_p^{\beta p} \tag{9}$$

where, Y is a dependent variable; $X_1, X_2,\cdots, X_p$ are independent variables; β_0 is the constant that the regression line intercepts the Y axis, representing the amount the dependent Y will be when all the explanatory variables are 0 and β_i, $1\leq i \leq p$ = the regression coefficient, representing the amount response variable Y changes when the explanatory variable changes 1 unit (Maason et al, 2003).

Regression estimation of the coefficients of an exponential model requires the use of logarithmic transformation applied to both sides of the equation. The results are:

$$\log(Y_i) = \log(\beta_0 X_1^{\beta1} X_2^{\beta2}\cdots X_p^{\beta p}) \Rightarrow$$
$$\log(Y_i) = \log(\beta_0) + \beta_1\log(X_1) + \beta_2\log(X_2) + \cdots + \beta_p\log(X_p) \tag{10}$$

The exponential function is linear in the logarithms of the dependent and independent variables. The coefficients β_1 and $\beta_2\cdots\beta_p$ are estimated directly in transformed regression. However, the transformed model constant, β_0, is the logarithm of the original equation constant. Thus the

antilogarithm must be computed (Carlson and Thorne, 1997).

Table 1 summarizes the range of experimental data used for construction of the objective new correlations.

Table 1 The range of experimental data

Property	Min	Max
Gas solubility (SCF/STB)	83	1708
Gas gravity	0.624	1.872
Oil gravity	0.554	0.858
API gravity	33.4	124
Saturated oil viscosity, CP	0.04	3
Reservoir temperature, ^{o}F	100	306

7 The new improved correlations (AUT Models)

In this study, all the independent variables were selected for the three dependent output variables, namely bubble point pressure (P_b), oil viscosity (μ_{ob}) and gas solubility (R_s). The logarithms Eq. 10 of all the variables were selected as inputs for analysis. Subsequently, a stepwise multiple linear regression analysis was carried out to determine the relationships between the dependent variable and the independent variables. To achieve this goal, regression analysis was carried out using the statistical software package SPSS for Windows. Stepwise regression procedures select the most correlated independent variable first, and then select the second independent variable which most correlates with the remaining variance in the dependent variable. This procedure continues until the selection of an additional independent variable does not increase the R-squared by a significant amount, usually a significance of at least 95%.

The most reliable and meaningful regression equations that could be obtained by the statistical analysis are given as equations 11, 12 and 13:

$$P_b = 107.93 R_s^{0.9129} \gamma_g^{-0.666} T^{0.2122} API^{-1.08} \tag{11}$$

$$\mu_{ob} = 6.28 \times 10^6 R_s^{-0.0266} T^{-1.032} API^{-2.786} \tag{12}$$

$$R_s = 0.0103 P_b^{1.014} \gamma_g^{0.719} T^{-0.223} API^{1.182} \tag{13}$$

8 Comparison between the AUT empirical equations and conventional models

The statistical significance and validity of the presently derived model was checked by using some test statistics. The R-squared, also called the coefficient of determination, is the proportion of variance in the dependent variable, which can be predicted from the independent variables. The R_2 value

calculated for the full models were R_2=0.93, 0.98 and 0.95 for P_b, μ_{ob} and R_s respectively. This indicates that 93 % of the variance of bubble point pressure, for example, can be predicted from variables T, R_s, γ_g and γ_o. R is the square root of R-squared and is the correlation between the observed and predicted values of dependent variable. The calculated correlation coefficient (R=0.93) indicates a strong relationship between the dependent and independent variables. The standard error of the estimate, also called the root mean square error, is the standard deviation of the error term, and is the square root of the mean square residual. The standard error of the estimate shows the standard deviation of the residuals to be 0.0610, 0.0548 and 0.0643 for P_b, μ_{ob} and R_s respectively.

For comparison purposes we used the models presented in sections 3 to 5. The results of different models including the new AUT model (present study) are given in Figs. 1, 3 and 5. These figures are each sorted versus the experimental corresponding property and show how the new equations follow the real data in comparison with other correlations in use, for bubble point pressure, saturated oil viscosity and gas solubility respectively.

The derived equations were then exposed to some 12 evaluation data. This was done by applying about 10% of the available data for evaluation and thereby the equations were qualified. We used 12 randomly selected sets of experimental data among the 106 sets of data that were available. These test data are also well scattered in the whole range of available data. The data used for the evaluation are given in Table 2.

Table 2 Data used for the evaluation

	P_b	Temp	R_s	Gas S. G	Oil S. G	API
1	400	219	123	0.835	0.8104	43.1
2	550	211	145	0.897	0.8139	42.4
3	650	290	251	0.755	0.6524	85.4
4	795	219	248	0.887	0.7903	47.5
5	855	200	223	0.722	0.7742	51.3
6	985	200	264	0.712	0.7733	51.5
7	1300	203	426	0.786	0.7592	54.9
8	1450	237	699	0.827	0.6584	83.4
9	1900	172	414	0.68	0.8134	42.5
10	2415	220	464	0.836	0.847	35.6
11	3000	185	811	0.812	0.842	36.6
12	4200	200	891	0.975	0.887	28.0

Figs. 2, 4 and 6 indicate that the proposed model could accurately predict P_b, μ_{ob}, and R_s respectively. These scatter plots also show a comparison with other aforementioned correlations.
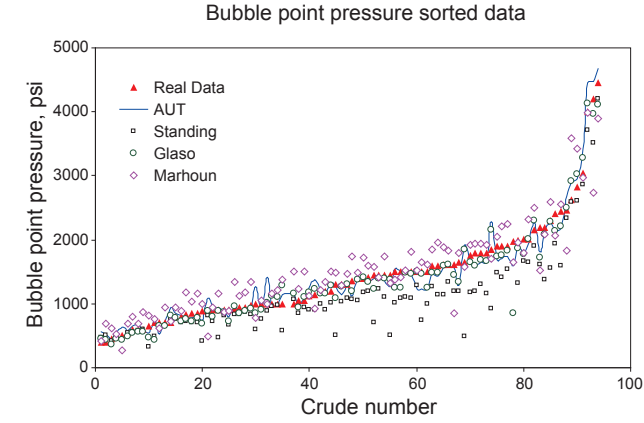

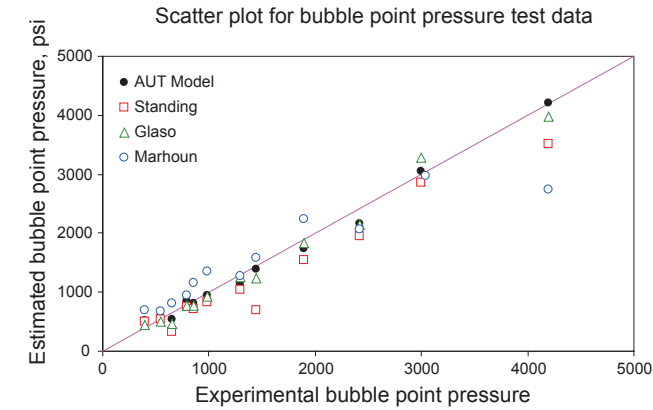**Fig. 1** Bubble point pressure vs. crude sample number

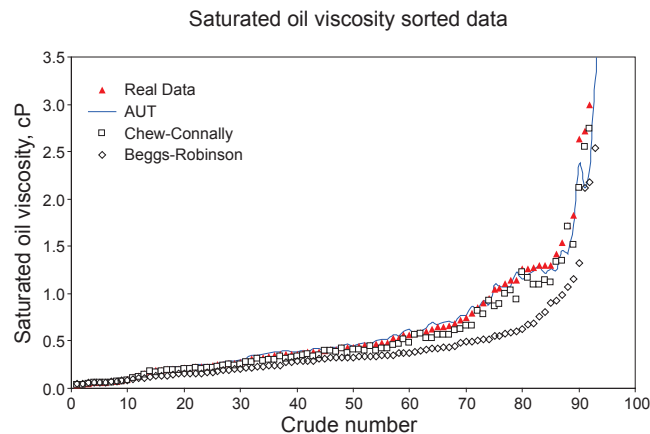**Fig. 2** Estimated vs. experimental P_b results for the testing data

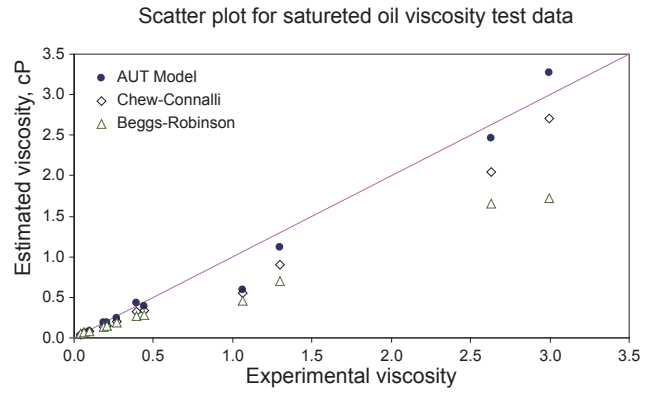**Fig. 3** Saturated oil viscosity vs. crude sample number

Fig. 4 Estimated vs. experimental μ_{ob} results for the testing data
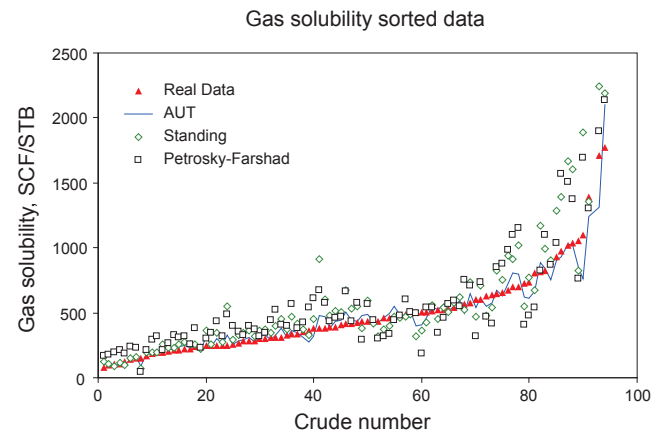

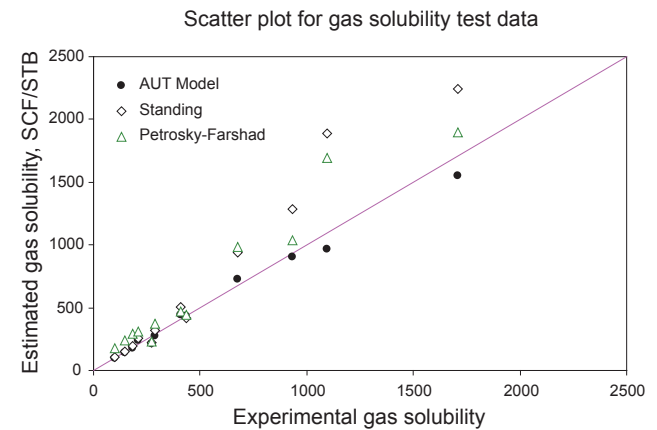**Fig. 5** Gas solubility vs. crude sample number

Fig. 6 Estimated vs. experimental R_s results for the testing data

9 Conclusions

In this study three simple to use, straight-forward empirical equations were proposed for estimation of bubble point pressure, saturated viscosity and gas solubility of crude oils respectively. The experimental data were correlated very well with the proposed correlations and the R^2 index results were

well near unity for all the bubble point pressure, saturated oil viscosity and gas solubility properties. While the reference data were extended over a wide span of magnitudes, the new models proved to be also of higher accuracy. This makes them more appropriate for applications compared to the conventional correlations.

Nomenclatures

API: American Petroleum Institute Index for oil gravity
P: Pressure, psia
P_b: Bubble point pressure, psia
R_s: Gas solubility or gas oil ratio [Standard cubic foot per standard condition barrel], SCF/STB
SC: Ambient standard conditions in terms of temperature and pressure
T: Temperature, ^{o}F
V_o: Oil volume
w: Connection weight
Greeks
γ_o: Oil gravity
γ_g: Gas gravity
μ_{od}: Dead oil viscosity
μ_{od}: Saturated oil viscosity
Subscripts
b: Bubble Point
g: Gas
o: Oil

s: Solubility

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