

## Review Paper

# Enhancement of NMR relaxation inversion: A review on pretreatment denoising

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## ABSTRACT

Nuclear magnetic resonance (NMR) is a sophisticated technology to gain insights into the Earth's physical and chemical properties, such as porosity, permeability, fluid viscosity, and pore structure in near-surface environments. The signal-to-noise ratio (SNR) is one of the most critical challenges in applying NMR to reservoir pore media, as the data acquired from NMR instruments must be inverted into NMR relaxation spectrum to estimate formation information, where the inversion is an inherently ill-posed problem. In particular, the target of NMR detection is transitioning to the ultra-deep reservoirs, which are characterized by an extremely low porosity. In these environments, the NMR data typically exhibit very low SNR due to the limited fluid volume within the sensitive region and harsh measurement conditions, both of which significantly impact the quality of the inverted spectra. Therefore, enhancing SNR prior to spectrum inversion, i.e., through data denoising, is essential.

This paper reviews methods for denoising NMR echo data, including mathematical transformation methods, morphological filtering techniques, and artificial intelligence (AI)-based methods. Their advantages and disadvantages of each method were compared and analyzed. The development trend in NMR data denoising is summarized. A multi-dimensional denoising strategy that integrates mathematical transformation and AI technologies, along with the development of lightweight AI models, shows great promise for NMR echo data denoising.

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## 1. Introduction

Nuclear magnetic resonance (NMR) is a highly sophisticated technique. As early as 1924, Pauli predicted from the fine structure of atomic spectra that certain atomic nuclei possess spin angular momentum and associated magnetic moments (Bloch et al., 1946a, 1946b, 1946c; Price, 2009; Purcell et al., 1946). In the presence of an external magnetic field, these magnetic moments generate

discrete energy levels. When exposed to radiofrequency (RF) radiation at a specific frequency, nuclei undergo resonance absorption transitions between these levels (Bloch et al., 1946b, 1946c; Purcell et al., 1946). In 1946, Purcell at Harvard University and Bloch at Stanford University independently discovered the NMR phenomenon (Purcell et al., 1946). Shortly thereafter, Russell Varian confirmed the precession of nuclear spins in the Earth's magnetic field.

Since its discovery, NMR has become an indispensable tool across diverse fields, including medical imaging, chemical analysis, and materials science (Ericsson and Hemmingsson, 1992; Hatzakis, 2019; Acri et al., 2020; Sebastiani et al., 2003; Smith and Strange, 1996; Hong and DeGrado, 2012; Coates et al., 1999). In medical imaging, Magnetic resonance imaging (MRI) is a non-invasive tomographic technique that provides high-contrast and high-resolution images of internal structures. Compared to the

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conventional X-ray imaging, MRI offers superior soft tissue contrast and avoids the use of ionizing radiation (Gao et al., 2024; Veetil et al., 2024; Matter et al., 2024; Braga et al., 2024). It is routinely applied in examination and diagnosis of the brain, spine, joints, chest, abdomen, and pelvis (Braga et al., 2024; Parnes et al., 2024).

The fundamental principle of NMR involves the interaction between nuclei spin systems and external static and RF magnetic fields. When a sample is placed in a strong static magnetic field, nuclear spins align along the field direction. An applied RF pulse excites the nuclei, causing transitions between energy states. As the nuclei return to equilibrium, they emit characteristic signals (Coates et al., 1999; Deng and Xie, 2010a). By analyzing how these signals evolve over time, it is possible to reconstruct information about the internal structure and physical properties of the sample.

In chemical and materials research, NMR is extensively used to determine molecular structures, investigate intermolecular interactions, and study material properties. Interpretation of NMR signals yields valuable parameters, such as chemical shift, coupling constants, and relaxation times, providing crucial insights for both fundamental research and applied engineering (Fulmer et al., 2010; Sebastiani et al., 2003).

The application of NMR in geoscience began in the 1950s, initially focusing on the properties of water- and oil-saturated rocks. By the 1990s, with the advancement of pulsed NMR logging instruments, the technique had gained widespread acceptance in the petroleum industry (Zhang and Zhang, 2021a; Ayorinde et al., 2020; Jiang et al., 2019; Liu et al., 2018). NMR logging utilizes the resonance phenomenon generated by the interaction between hydrogen nuclei and a magnetic field to investigate subsurface oil and gas reservoirs, providing an effective method for characterizing rock physical properties (Liu et al., 2018; Jin et al., 2019; Guo et al., 2020).

NMR logging directly probes the hydrogen nuclei of pore fluids within formations. The inverted  $T_2$  spectrum derived from NMR data enables the evaluation of key petrophysical parameters, such as porosity, permeability, and fluid saturation, highlighting the broad application potential of NMR logging in oil and gas reservoir exploration and development (Elsayed et al., 2022; Casabianca et al., 2014; Dunn and LaTorraca, 1999; Heidary, 2021; Valle et al., 2021; Hu et al., 2016; Cai et al., 2022).

NMR logging typically involves the measurement the longitudinal relaxation time  $T_1$  and transverse relaxation time  $T_2$  in fluid-saturated rocks (Xu and Zhang, 2023; Liu et al., 2022; Gu et al., 2023). In practical,  $T_2$  measurements are more commonly used due to their shorter acquisition time. The echo train data collected by NMR logging tools exhibit multi-exponential decay behavior, and the data discrete form at complete polarization can be expressed as (Guo et al., 2019a; Xu et al., 2023):

$$M_i = \sum_{j=1}^n f_j e^{-t/T_{2j}} + \varepsilon_i \quad (1)$$

where  $M_i$  is the signal magnitude of the  $i$ th echo,  $t$  is the acquisition time,  $T_{2j}$  is the  $j$ th preset  $T_2$  value,  $f_j$  is the corresponding magnitude at  $T_{2j}$ ,  $n$  is the number of preset  $T_2$  values, and  $\varepsilon_i$  represents noise in the  $i$ th measurement.

Eq. (1) can be written in matrix form as:

$$\mathbf{M} = \mathbf{A}\mathbf{f} + \mathbf{\varepsilon} \quad (2)$$

where  $\mathbf{M}$  is the vector of measured echo data,  $\mathbf{A}$  is the  $m \times n$  kernel matrix,  $\mathbf{f}$  is the  $T_2$  distribution spectrum to be solved, and  $\mathbf{\varepsilon}$  is the noise vector.

NMR data processing aims to transform the collected echo data into an intuitive  $T_2$  spectrum. In other words, given Eq. (2) with known  $\mathbf{M}$  and  $\mathbf{A}$ , the objective is to solve for  $\mathbf{f}$ . This constitutes an ill-posed inverse problem, where noise significantly affects the accuracy of the inverted  $T_2$  spectrum (Zou et al., 2016a, 2018). A lower signal-to-noise ratio (SNR) results in poorer reliability of the inverted spectrum (Song et al., 2002; Venkataramanan et al., 2002).

In NMR logging, the echo data typically exhibit a low SNR, which can be attributed to several factors (Gupta et al., 2019). First, the degree of polarization of the NMR instrument influences signal strength. Additionally, complex noise sources within the formation, such as magnetic heterogeneity in rocks and impurities, can degrade signal reception and distort the echo. Moreover, during signal transmission, factors such as absorption, scattering, and attenuation by the formation medium further reduce signal strength, thereby lowering the SNR.

To enhance the SNR of NMR logging echo data, several strategies may be employed. These include using optimized pulse sequences and acquisition parameters, such as extended sampling times and multiple scans, along with advanced signal processing techniques (Song and Kausik, 2019). Adaptive filtering is one of the most commonly used denoising methods prior to echo detection (Liao et al., 2021). This method dynamically adjusts filter weights based on the statistical characteristics of the signal, enabling real-time noise suppression. Additionally, digital phase-sensitive detection can improve the SNR by enhancing signal amplitude through phase measurement, and it has been widely applied in modern NMR instrumentation (Luo et al., 2022a).

Denoising after echo detection is currently the most common and cost-effective method for improving the SNR of NMR echo data (Koehl, 1999). To improve the accuracy of  $T_2$  spectrum inversion and enhance the reliability of NMR applications in unconventional oil and gas reservoirs, it is critical to apply effective denoising prior to inversion.

This study aims to systematically review denoising methods for NMR echo data developed over recent decades, classifying and analyzing their characteristics, principles, and application scenarios. We focus on three categories: (1) mathematical transformation methods, (2) morphological filtering methods, and (3) artificial intelligence (AI)-based methods. Representative techniques from each category are discussed in detail, including their theoretical foundations, implementation, strengths, and limitations. The main contribution of this paper is to provide a clear categorization framework and trace the technological evolution of NMR echo data denoising, offering valuable reference points for future research.

The paper is organized as fourth parts. Section 1 emphasizes the central role of NMR logging in oil and gas exploration and development. With increasing demand for precise reservoir characterization, the importance of NMR echo data denoising is growing. A comprehensive review of the research background reveals the limitations of existing denoising methods under complex noise environments and low SNR conditions, highlighting the need for more systematic and robust approaches.

Section 2 introduces the complete processing workflow for NMR echo data, including phase rotation, denoising, data compression, and spectrum inversion, to help readers understand the overall structure of NMR data analysis.

Section 3 presents a systematic review of various denoising techniques. For instance, mathematical transformation methods offer strong theoretical support and are easy to implement but have limited effectiveness at low SNR. Morphological filtering methods perform well in cases where noise exhibits strong

structural characteristics but are highly sensitive to parameter selection. AI-based methods offer high adaptability and robustness in complex environments but often require large, high-quality training datasets.

Section 4 provides a comprehensive summary and outlook. This includes a multi-dimensional perspective on current denoising approaches and identifies future directions. Specifically, research is expected to focus on innovations in physics-informed AI denoising, as well as hybrid methods that combine traditional signal processing techniques with AI models. These directions aim to further advance the effectiveness, adaptability, and efficiency of NMR echo data denoising in challenging subsurface environments.

## 2. NMR data processing procedure

Raw NMR data consist of both real and imaginary components. Echo signals are obtained through phase rotation of these components and exhibit multi-exponential decay behavior. The data are then subjected to preprocessing steps, such as denoising and compression, followed by inversion to generate the NMR  $T_2$  spectrum (Lascos and Cassidy, 2008). Mathematical analysis of the  $T_2$  spectrum provides valuable insights into the petrophysical and fluid properties of the reservoir (Guo et al., 2023).

The actual NMR logging data workflow in the wellbore contains data acquisition, phase rotation, data denoising, data compression, and spectra inversion, as shown in Fig. 1. Initially, the NMR instrument is deployed downhole to collect data comprising real and imaginary components. The raw echo signals are derived via phase rotation. The steps highlighted in the magenta box in Fig. 1 represent the core components of the NMR data processing workflow. In this paper, we focus specifically on reviewing the denoising stage within these core steps.

In NMR logging, signal phase shifts may occur due to various factors, such as the complex downhole environment, magnetic field inhomogeneity, and instrument performance. These phase shifts can lead to inaccurate projections of the real and imaginary components of the signal, thereby affecting the accuracy of subsequent inversion processes (Guo et al., 2023). Phase rotation techniques address this issue by calculating the initial phase shift angle and applying a phase rotation factor to perform a rotational transformation. This ensures that the real component of the signal contains valid echo decay information, while the imaginary component approaches zero (Takahashi, 2022).

Specifically, the initial phase shift angle can be determined using the maximum peak method, the zero-crossing method, or the frequency-domain analysis. The phase is then corrected using the rotation factor, resulting in a real component that accurately reflects the echo decay characteristics while minimizing noise in the imaginary component. This technique effectively enhances the SNR, reduces phase errors, and improves inversion accuracy, thus providing reliable data support for subsequent  $T_2$  spectrum inversion and reservoir evaluation. After phase rotation, however, the echo data still contain noise originating from multiple down-hole sources.

In downhole NMR measurements, the acquired echo data inevitably contain noise originating from both the measurement system and the complex subsurface environment (Xiao, 2016). The dominant component is Gaussian white noise from the receiving electronics, while additional fluctuations arise from hardware instability, gradient field imperfections, temperature drift, and other tool-related influences (Bloembergen et al., 1948; Menger and Prammer, 1998). These effects are further amplified under downhole conditions. Mechanical vibrations, telemetry interference, and large temperature–pressure variations can all perturb the echo decay, making the signal more susceptible to distortion. Moreover, reservoir heterogeneity, such as variations in magnetic susceptibility, pore structure, and fluid mobility, may alter the local magnetic field and lead to phase inconsistency or deviations in the expected attenuation pattern. When the SNR is low, the combined impact of these noise sources becomes particularly noticeable, often causing significant variability in the early echoes that are crucial for accurate  $T_2$  spectrum inversion (Li et al., 2022a; Wang et al., 2023). Therefore, effectively identifying and mitigating the dominant noise components is essential for preserving the integrity of the echo response and ensuring reliable denoising, inversion, and reservoir characterization.

NMR instruments typically acquire echo signals at a high sampling rate. However, in certain application scenarios, the high sampling rate leads to data redundancy. In such cases, data dimensionality reduction becomes essential. The core principle of echo data compression is to exploit the inherent redundancy and characteristic distribution of the echo signal. By applying specific mathematical transformations, the original high-dimensional data can be mapped into a lower-dimensional space, significantly reducing data volume while preserving essential signal information as much as possible (Menger and Prammer, 1998).

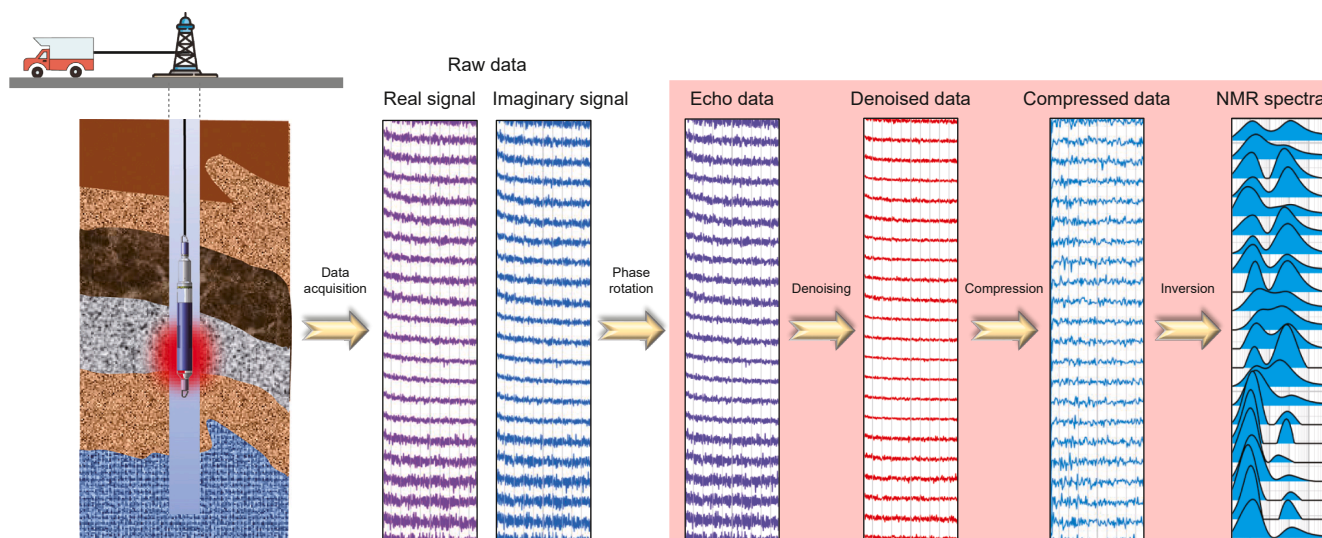


Fig. 1. Workflow diagram of downhole NMR log data processing.



Currently, commonly used data compression methods in NMR echo data processing include principal component analysis (PCA), the window averaging method, truncated singular value decomposition (TSVD), fast singular value decomposition (FSVD) method, and hybrid approaches (O'Neill et al., 2022; Sun et al., 2021; Ding et al., 2016a; Guo et al., 2019b). For example, PCA is a widely used feature extraction method that employs linear transformation to project the original data onto directions of maximum variance. This process not only reduces dimensionality but also effectively extracts principal components, thereby retaining critical signal information.

During data compression, it is crucial to balance the trade-off between information retention and compression ratio. Excessive compression may lead to the loss of important signal features, whereas insufficient compression may fail to reduce computational complexity, limiting its practical utility. Therefore, careful selection of compression methods and optimization of relevant parameters play a decisive role in enhancing the efficiency and accuracy of subsequent inversion analysis. For readers interested in a more in-depth exploration of data compression techniques, including detailed comparisons of various algorithms, practical case studies, and applications in emerging research areas, please refer to references (Li et al., 2022a; Wang et al., 2023; O'Neill et al., 2022; Sun et al., 2021; Ding et al., 2016a; Guo et al., 2019b).

After compression, the echo data undergo inversion processing. NMR data inversion is a severely ill-posed problem, making the development of fast, stable, and high-precision inversion methods essential for obtaining accurate NMR spectra that meet the real-time processing requirements of NMR logging (Ding et al., 2016a; Guo et al., 2019a).

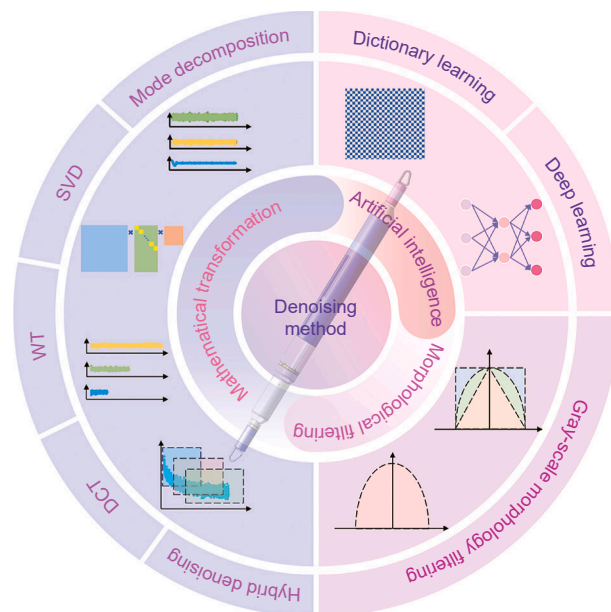
Currently, the commonly used inversion techniques can be broadly categorized into two groups: iterative methods and regularization-based methods (Gu et al., 2021a; Zou and Xie, 2016). Iterative methods, such as non-negative least squares (NNLS) and singular value decomposition (SVD)-based approaches, approximate the target solution by progressively refining an initial estimate (Guo et al., 2018a, 2019c). While computationally simple, these methods are often sensitive to noise interference and the choice of initial parameters.

In contrast, regularization-based methods enhance inversion stability and physical consistency by incorporating regularization terms into the objective function, thereby suppressing solution instability (Ding et al., 2016b; Gu et al., 2021a; Zou et al., 2016b). Due to their robustness and flexibility, regularization methods have become the most widely used inversion techniques in NMR data processing. For readers seeking a deeper understanding of NMR data inversion, including comparisons of different algorithms, case studies, and applications in new fields, please consult references (Gu et al., 2021a; Zou and Xie, 2015, 2016; Guo et al., 2018a, 2019c; Cobas et al., 2004; Zou et al., 2016b).

### 3. NMR data denoising methods

Denoising is a key step in NMR data processing, aiming to reduce noise interference with structural information and to improve the accuracy and interpretability of the data (Zou et al., 2016b). Currently, denoising methods for NMR echo data can be broadly categorized into three groups, as illustrated in Fig. 2.

The first category comprises mathematical transformation-based methods, which can be further divided into five main types. The first type is the SVD-based methods. These approaches decompose the data matrix and retain components with larger singular values, effectively removing noise and reducing the ill-posedness of echo data inversion (Berman et al., 2013; Casabianca et al., 2014). The



**Fig. 2.** Classification of current NMR echo data denoising methods. DCT, discrete cosine transformation; WT, wavelet transformation; SVD, singular value decomposition.

second type involves the mode decomposition-based methods, including empirical mode decomposition (EMD), symplectic geometry mode decomposition (SGMD), and variational mode decomposition (VMD) (Bloembergen et al., 1948; Gu et al., 2021b; Guo et al., 2018b, 2023; Guo and Xie, 2018; Koehl, 1999; Zhou et al., 2025). These techniques adaptively suppress noise by decomposing the signal and selectively reconstructing its components, thereby enhancing data quality across different signal characteristics and noise distributions. The third type is the wavelet transformation (WT)-based methods. Leveraging multi-scale and multi-resolution properties, these methods decompose signals into sub-signals at different frequencies and scales. Techniques such as thresholding, adaptive filtering, and regularization with heuristic thresholding are employed to achieve denoising, significantly improving the quality of low-SNR data (Deng and Xie, 2010a; Chen et al., 1994; Wu and Huang, 2010; Cai et al., 2010; Zhou et al., 2025; Mallat and Hwang, 2002; Zheng and Zhang, 2007a; Xie et al., 2014; Meng et al., 2015a; Trbovic et al., 2005; Cancino-De-Greiff et al., 2002). The fourth type is discrete cosine transformation (DCT)-based methods. These approaches apply variable window lengths to denoise the initial echoes, aiming to optimize the overall denoising effect (Gu et al., 2021c). The fifth type is hybrid denoising methods, which integrate the strengths of various individual denoising approaches to achieve enhanced performance (Zhao et al., 2025).

The second category includes morphological filtering methods, which exploit morphological differences between signals and noise. For example, morphological filtering has been effective in suppressing the dispersion of linear coherent noise in seismic exploration. Multi-scale mathematical morphology methods, widely used for signal denoising and feature extraction, offer high robustness and accuracy (Wu et al., 2011; Xiao et al., 2013). Additionally, the grayscale morphological filtering improves denoising performance for low-frequency and random noise by using optimized structural elements such as elliptical structuring elements (Gao et al., 2021). These techniques exhibit strong stability and adaptability when applied to NMR log data with varying  $T_2$  spectra.

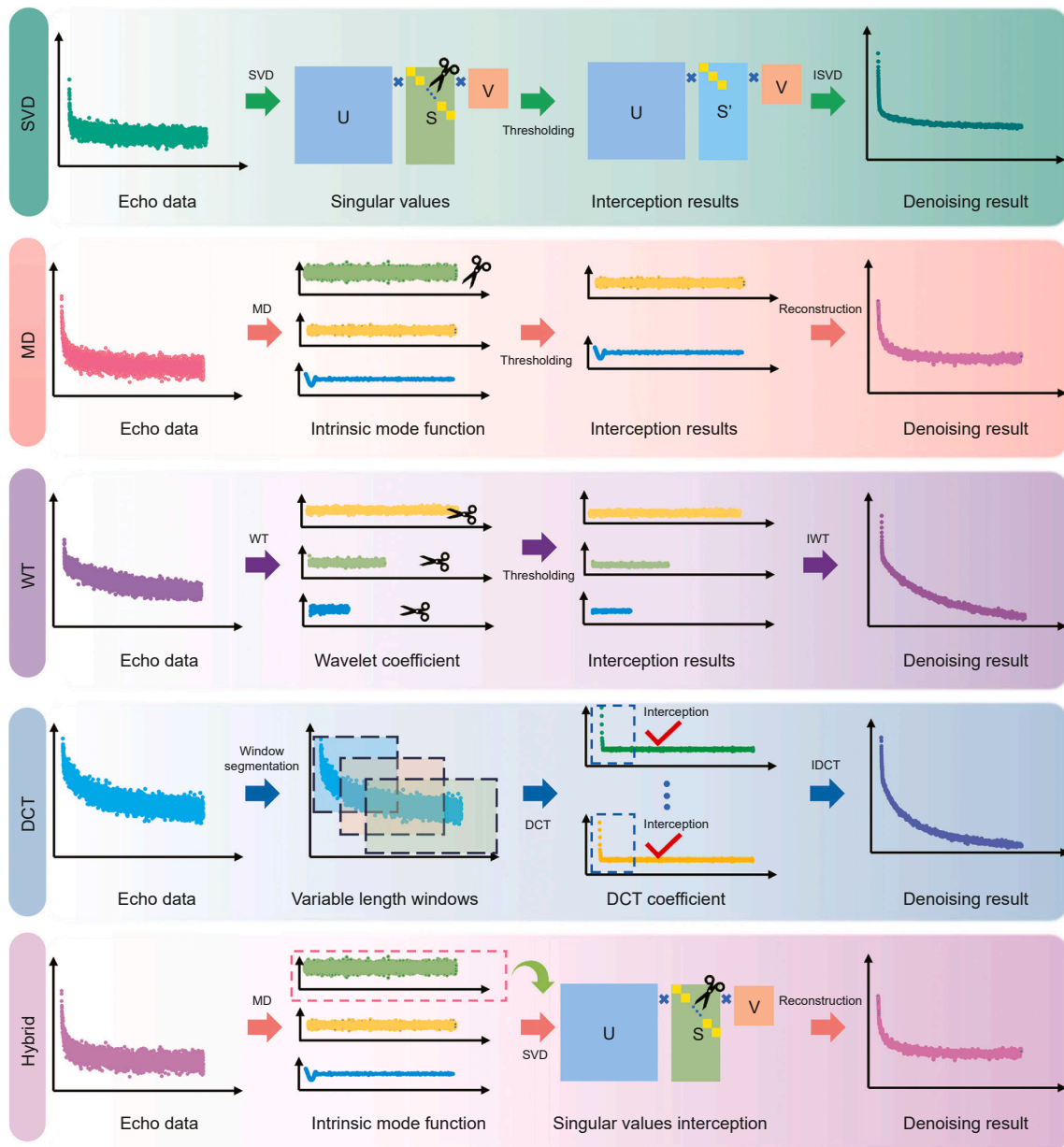


The third category encompasses AI-based methods, which utilize machine learning or deep learning techniques for adaptive modeling and echo data processing. For instance, dictionary learning, based on sparse representation theory, constructs over-complete dictionaries and sparse coefficients from the training datasets (Luo et al., 2022b). This enables adaptive extraction and reconstruction of useful signal information, thereby improving data quality and inversion stability. Deep learning methods employ network models to fit logging data and generate simulated noise datasets, which are then used to train multi-scale echo denoising networks (MsEDNet) (Liu et al., 2023). These networks learn signal features and optimal transformation spaces adaptively, achieving efficient and robust denoising of NMR logging data. By leveraging the powerful learning capabilities of AI, these methods demonstrate excellent performance in noise reduction and enhancement of NMR echo data quality.

### 3.1. Mathematical transformation method

The mathematical transformation methods represent a broad and powerful class of techniques. Their core principle involves applying specific mathematical operations to convert raw NMR echo data from one domain or representation to another. This transformation reveals underlying features, structures, or patterns within the data, thereby enabling more effective noise suppression and signal enhancement (Huang et al., 2017). These methods not only improve the SNR but also facilitate subsequent processing tasks such as inversion and interpretation.

Fig. 3 illustrates schematic diagrams of the denoising processes for NMR echo data using five representative mathematical transformation approaches. Each method applies a distinct transformation strategy, such as matrix decomposition, frequency domain manipulation, or multi-scale analysis, to isolate and



**Fig. 3.** Schematic diagrams of the denoising processes for NMR echo data with the mathematical transformation methods. SVD, singular value decomposition; MD, mode decomposition; WT, wavelet transformation; DCT, discrete cosine transformation; ISVD, IWT, and IDCT, inverses;  $U$ ,  $S$ ,  $S'$ ,  $V$ , the three matrices representing singular values.

suppress noise components while preserving essential signal characteristics.

From a general principle, mathematical transformation methods are primarily based on signal characteristic analysis across various domains, such as time domain, frequency domain, and time-frequency domain. In NMR logging, echo signals are often affected by multiple sources of interference, including instrumental noise and formation-related environmental noise. Notably, the distributions of noise and useful signals tend to differ across these domains. For instance, in the frequency domain, signal energy is typically concentrated within specific frequency bands, while noise is more broadly or differently distributed. Mathematical transformations allow the mapping of signals into these alternative domains, enabling the separation of signal and noise components and thereby facilitating more effective noise identification and suppression.

Historically, early mathematical transformation methods for NMR echo data denoising were relatively simple and mainly relied on classical approaches such as the Fourier transform. However, with advances in our understanding of signal characteristics and the rapid development of computational techniques, more sophisticated transformation methods have been introduced. For instance, the advent of wavelet transforms has addressed the limitations of Fourier analysis in time-frequency localization, allowing better detection and processing of transient features and fine details in NMR signals, thereby significantly improving denoising performance.

In recent years, research has increasingly focused on the integration of multiple mathematical transformation techniques. By leveraging the complementary strengths of various methods, hybrid denoising algorithms have been developed to address the challenges posed by increasingly complex and variable noise environments. Additionally, with the rise of AI, new methods have emerged that integrate mathematical transformations with machine learning and deep learning algorithms. These hybrid frameworks enable data-driven learning of the relationships between transformed features and noise characteristics, leading to more intelligent, adaptive, and precise denoising outcomes.

### 3.1.1. Singular value decomposition (SVD)

A key application of SVD is the separation of signal and noise (Guo et al., 2016). In practical NMR data denoising, SVD leverages its unique matrix decomposition properties. The echo data acquired in NMR logging can be arranged into a matrix, which is then decomposed into the product of three matrices. The diagonal elements of one of these matrices represent the singular values, typically ordered in descending magnitude. Larger singular values correspond to the primary signal components, as signals usually concentrate most of the energy in the data, whereas smaller singular values are associated with noise due to its lower energy content (Francischello et al., 2021).

SVD achieves optimal data truncation by retaining the first  $k$  singular values and their corresponding singular vectors, effectively functioning as denoising operation by discarding low-amplitude noise components and thereby improving the SNR.

However, the complex and non-stationary nature of NMR signals presents challenges for conventional SVD-based denoising. NMR echo signals often exhibit temporal locality, manifested as patterns or periodicities within the time domain (Wu and Huang, 2010), which can be overlooked by standard SVD. To address this, constructing a Hankel matrix from the original data prior to SVD application has proven to be effective. A Hankel matrix, formed using time-shifted subsequences of the data, better captures local temporal features in the signal and enhances the effectiveness of the subsequent decomposition.

The singular values of a Hankel matrix typically show a sharp decay, allowing a low-rank approximation by retaining only the dominant singular values and their vectors. This enables effective noise suppression while preserving signal integrity.

The Hankel matrix  $\mathbf{H}$  is constructed from the echo data  $M$  as follows:

$$\mathbf{H} = \begin{bmatrix} M_1 & M_2 & \dots & M_n \\ M_2 & M_3 & \dots & M_{n+1} \\ \dots & \dots & \dots & \dots \\ M_C & M_{C+1} & \dots & M_{C+n-1} \end{bmatrix} \quad (3)$$

where  $M_i$  is the amplitude of the  $i$ th echo,  $i = 1, 2, \dots, m$ , and  $m = aC$ .  $m$  is the echo number,  $a = 2$ .  $C$  is a data point in the echo data, from which the Hankel matrix is filled according to the sliding window rule. The rank of  $\mathbf{H}$  is  $r = \min(C, n)$ . The SVD of  $\mathbf{H}$  is given by:

$$\mathbf{H} = \mathbf{U}\mathbf{S}\mathbf{V}^T \quad (4)$$

$$\mathbf{S} = \text{diag}(\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_r) \quad (5)$$

where  $\mathbf{U}$  and  $\mathbf{V}$  are orthonormal matrices, and  $\mathbf{V}^T$  is the transpose of  $\mathbf{V}$ .  $\mathbf{S}$  is a diagonal matrix containing singular values  $\sigma_1 > \sigma_2 > \sigma_3 > \dots > \sigma_r > 0$ .

A hard thresholding method is applied to the singular values, retaining only those greater than a predefined threshold  $\sigma_{\text{th}}$ . The noise-related components, which correspond to smaller singular values, are set to zero. This yields a new diagonal matrix  $\mathbf{S}'$ . Substituting  $\mathbf{S}'$  into Eq. (4) produces a new Hankel matrix, from which the denoised signal is reconstructed. This truncated SVD process effectively removes noise while preserving key signal components.

However, in standard SVD denoising, the decomposition operates globally, limiting its ability to effectively capture local signal features and leading to potential loss of detail. Randomized singular value decomposition (rSVD) addresses this limitation by introducing random projection techniques, offering better adaptability to complex and diverse noise environments compared to conventional SVD. It is more flexible in handling noise with varying distribution characteristics and can balance both global and local signal features, thereby retaining useful information in the echo data more effectively during the denoising process.

In rSVD, dimensionality reduction is first performed on the Hankel matrix  $\mathbf{H}$ , constructed from the echo data  $M$ , to obtain a reduced matrix  $\mathbf{H}_1$ :

$$\mathbf{H}_1 = \mathbf{H}\mathbf{Y} \quad (6)$$

where the matrix  $\mathbf{Y}$  is an  $n \times P$  random matrix whose elements follow a Gaussian distribution. The parameter  $P$  satisfies  $K_1 \leq P \ll n$ , where  $K_1$  represents the estimated number of signal components, and  $n$  is the number of columns in  $\mathbf{H}$ . This random projection reduces the original matrix  $\mathbf{H} \in \mathbf{R}^{m \times n}$  to a smaller matrix  $\mathbf{H}_1 \in \mathbf{R}^{m \times P}$ .

SVD is then applied to  $\mathbf{H}_1$ , yielding an orthogonal matrix  $\mathbf{U}_1$ , which serves as an approximate estimate of the left singular matrix  $\mathbf{U}$  from the full SVD of  $\mathbf{H}$ . By substituting  $\mathbf{U}_1$  with  $\mathbf{U}$ , a denoised matrix  $\mathbf{H}_2$  is reconstructed. The denoised signal is subsequently obtained via reverse diagonal averaging. The rSVD method effectively suppresses most noise components. However, the reconstructed signal may exhibit a reduction in amplitude.

Fig. 4 presents the denoising results of NMR echo data using both the SVD and rSVD methods at an SNR of 20. As shown, both methods significantly reduced noise, and the resulting  $T_2$  spectra show improved inversion accuracy. Notably, the  $T_2$  spectrum obtained via rSVD is closest to the original synthetic model. Although

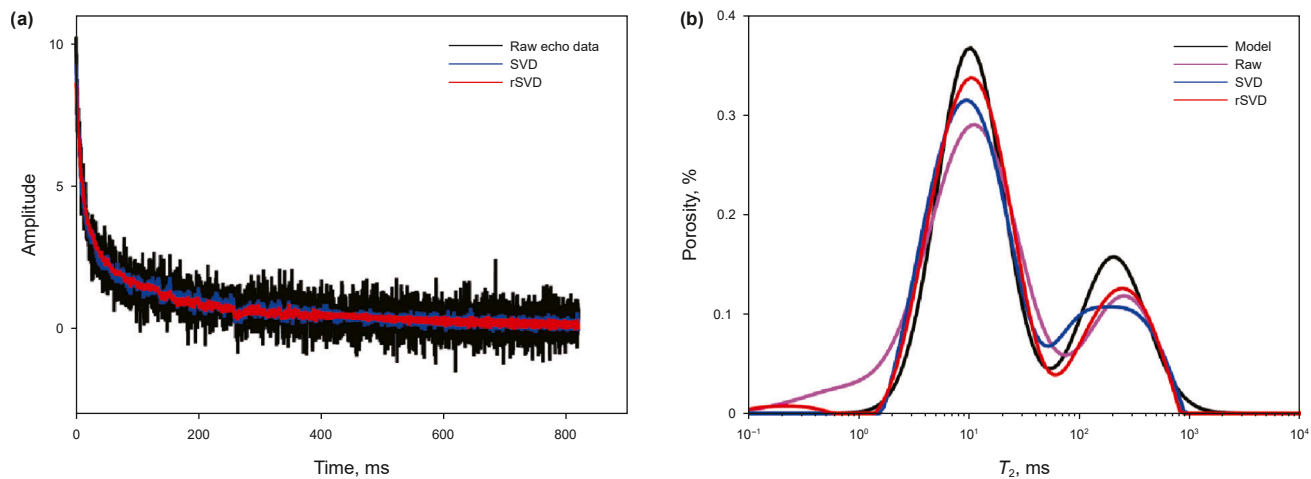


Fig. 4. Echo data (a) and corresponding inverted  $T_2$  spectra (b) with and without denoising using the SVD and rSVD methods.

both techniques enhanced inversion results, the long-relaxation components remain suboptimal due to limitations in the thresholding approach.

The selection of the truncation threshold plays a critical role in distinguishing signal from noise. In the current case, the setting threshold failed to adequately suppress noise in the long-relaxation region and led to the loss of some useful signal components, thereby affecting inversion accuracy. Future research should be focused on developing a more scientifically grounded and adaptive thresholding strategy to achieve more precise signal-noise separation, ultimately enhancing the denoising effectiveness and inversion reliability of NMR echo data.

3.1.2. Mode decomposition

The core objective of mode decomposition is to break down complex non-linear and non-stationary signals into a series of relatively simple and physically meaningful intrinsic mode functions (IMFs). These IMFs reveal the inherent features and varying patterns of the original signal more clearly, providing a solid foundation for subsequent signal analysis and feature extraction tasks.

In the early stages, researchers primarily relied on traditional linear and stationary signal processing methods for mode decomposition. However, as practical applications increasingly encountered vast amounts of nonlinear and non-stationary signals, the limitations of these traditional methods became apparent. They struggle to address the complex and variable demands of real-world scenarios.

To address these challenges, a variety of advanced mode decomposition algorithms have been developed. Among them, the EMD has received extensive attention due to its adaptive framework (Rilling et al., 2003; Yang et al., 2022). Subsequently, the VMD introduced a more rigorous optimization-based formulation, significantly improving decomposition robustness and mode separation accuracy (Koehl, 1999). In addition, the SGMD has emerged as a novel method capable of capturing intrinsic

oscillatory patterns based on symplectic geometric structures, expanding the toolset for analyzing nonlinear and non-stationary geophysical signals (Zhou et al., 2025). Comparative studies have shown that, while these methods are all effective to varying degrees in handling NMR signals, the VMD often demonstrates superior stability, clearer mode separation, and improved denoising performance. Therefore, in this work, we primarily focus on the VMD technique and provide a detailed discussion of its principles and applications in NMR echo denoising. Table 1 summarizes the comparative advantages and limitations of the EMD, SGMD, and VMD method, providing a concise reference for selecting suitable mode decomposition methods under different NMR signal conditions.

Given its superior robustness and clear mode separation, VMD is selected as the primary focus of this work. VMD is an advanced signal processing technique founded on variational principles, offering a novel and systematic approach to signal decomposition. Unlike traditional EMD, the VMD reformulates the complex problem of signal decomposition as a variational optimization problem (Chen et al., 2001; Guo et al., 2023). The core concept of VMD is the assumption that each mode is a band-limited signal with a specific central frequency and bandwidth. In practical applications, VMD constructs a carefully designed variational model to adaptively segment signals in the frequency domain. This allows VMD to decompose a signal into distinct modal components based on inherent frequency-domain characteristics, while ensuring maximal independence among these components in the frequency spectrum.

Importantly, after decomposition, these modal components can be seamlessly reconstructed to restore the original signal, preserving the integrity of the signal information. The implementation of VMD relies on an iterative optimization process that continuously updates key parameters, such as the central frequency and bandwidth of each mode. During each iteration, the algorithm refines these parameters according to the current signal state and the characteristics of each mode, driving the variational objective

Table 1  
Advantages and limitations of the mode decomposition denoising methods.

| Method | Advantages                                                                 | Limitations                                          |
|--------|----------------------------------------------------------------------------|------------------------------------------------------|
| EMD    | Adaptive decomposition, suitable for nonlinear signals                     | Mode mixing, sensitivity to noise, limited stability |
| SGMD   | Preserves intrinsic signal structure, suppresses mode confusion            | High computational cost, excessive component number  |
| VMD    | Clear mode separation; robust optimization; strong denoising under low SNR | Reduced adaptivity                                   |



function towards its minimum. When convergence is reached, i.e., when the objective function achieves its minimum value, the optimal decomposition is obtained.

This global variational optimization approach allows VMD to overcome several inherent limitations of EMD. For instance, EMD often suffers from inaccurate envelope interpolation due to the improper selection of local extrema, leading to under- or over-envelope effects that compromise decomposition accuracy (Liu et al., 2023; Jun and Leslie, 2017; Guo et al., 2016; Francischello et al., 2021; Gavish and Donoho, 2014). In contrast, VMD mitigates these issues through its robust, global optimization framework, enabling more reliable and accurate signal decomposition (Guo et al., 2023).

During the VMD process of the NMR signal  $M(t)$ , the signal is decomposed into multiple band-limited components in the frequency domain. VMD is implemented by solving a constrained variational problem, which is expressed as follows:

$$\begin{cases} \min_{\{m_k\}, \{\omega_k\}} \left\{ \sum_{k=1}^K \left\| \partial_t \left\{ \left[ \left( \delta(t) + \frac{j}{\pi t} \right) \cdot m_k(t) \right] e^{-j\omega_k t} \right\} \right\|_2^2 \right\} \\ \text{s.t. } \sum_{k=1}^K m_k(t) = M(t) \end{cases} \quad (7)$$

where  $m_k(t)$  is the  $k$ th IMF to be decomposed, and  $\omega_k$  is the corresponding center frequency.  $\delta(t)$  is the Dirac function,  $M(t)$  is the acquired NMR echo data, and  $K$  is the predetermined number of IMF. Since VMD does not require the calculation of upper and lower envelopes, it effectively avoids endpoint effects.

To solve the constrained variational problem, the Lagrange multiplier  $\lambda(t)$  and a quadratic penalty term  $\alpha$  are introduced, transforming the constrained problem into an unconstrained one. Consequently, Eq. (8) is reformulated into the augmented Lagrangian function:

$$\begin{aligned} \mathcal{L}(\{m_k\}, \{\omega_k\}, \lambda) = & \alpha \sum_{k=1}^K \left\| \partial_t \left[ \left( \delta(t) + \frac{j}{\pi t} \right) \cdot m_k(t) \right] e^{-j\omega_k t} \right\|_2^2 \\ & + \langle \lambda(t), M(t) - \sum_{k=1}^K m_k(t) \rangle + \left\| M(t) - \sum_{k=1}^K m_k(t) \right\|_2^2 \end{aligned} \quad (8)$$

The alternating direction method of multipliers (ADMM) is employed for iterative optimization, and the problem is transformed into the frequency domain. The convergence condition for iteration is:

$$\sum_{k=1}^K \frac{\left\| \hat{m}_k^{n+1} - \hat{m}_k^n \right\|_2^2}{\left\| m_k^n \right\|_2^2} < \zeta \quad (9)$$

where  $\zeta$  is the predefined threshold.

A critical parameter in the VMD method for decomposing NMR echo data is the number of IMFs, denoted as  $K$ . The effectiveness of VMD-based NMR echo data denoising depends on the appropriate selection of IMFs. It is crucial to determine an optimal  $K$  value before applying VMD and to identify the effective IMFs after decomposition to achieve optimal denoising results.

Guo et al. (2023) proposed a method for adaptively choosing the optimal  $K$ -value and the number of effective IMFs based on the

absolute difference between neighbouring squared errors (Koehl, 1999). The process involves setting different  $K$  values ( $1 \leq K \leq 10$ ) and performing VMD decomposition for each  $K$ . The absolute difference in squared errors between  $K$  and  $K-1$  is calculated and denoted as  $\delta_K$ . The  $K$  value corresponding to the largest  $\delta_K$  is chosen as the optimal IMF count,  $K_{\text{opt}}$ . All IMFs are then derived by performing VMD with  $K_{\text{opt}}$ , and the correlation coefficient  $R_i$  between the  $i$ th IMF ( $1 \leq i \leq K_{\text{opt}}$ ) and the original echo data is calculated as:

$$R_i = \frac{\text{Cov}(I_i, M)}{\sqrt{\text{Var}[I_i] \text{Var}[M]}} \quad (10)$$

where  $\text{Cov}(I_i, M)$  represents the covariance between  $I_i$  and  $M$ ,  $\text{Var}[I_i]$  is the variance of  $I_i$ ,  $I_i$  is the  $i$ th IMF, and  $M$  is the original echo data.

Since the frequencies of the IMFs decreases progressively from the first to the  $K$ th IMF, the correlation coefficient  $R_i$  between the  $i$ th IMF and the original echo data gradually increases with  $i$ . The correlation coefficient  $R_i$  is influenced by the noise level of the signal. The effective IMFs are determined adaptively using the mean value of all  $R_i$ . Specifically, when  $R_i > r$ , where  $r$  is the mean value of all  $R_i$ , the corresponding IMF is considered part of the effective signal. The sum of these effective IMFs constitutes the denoised echo data.

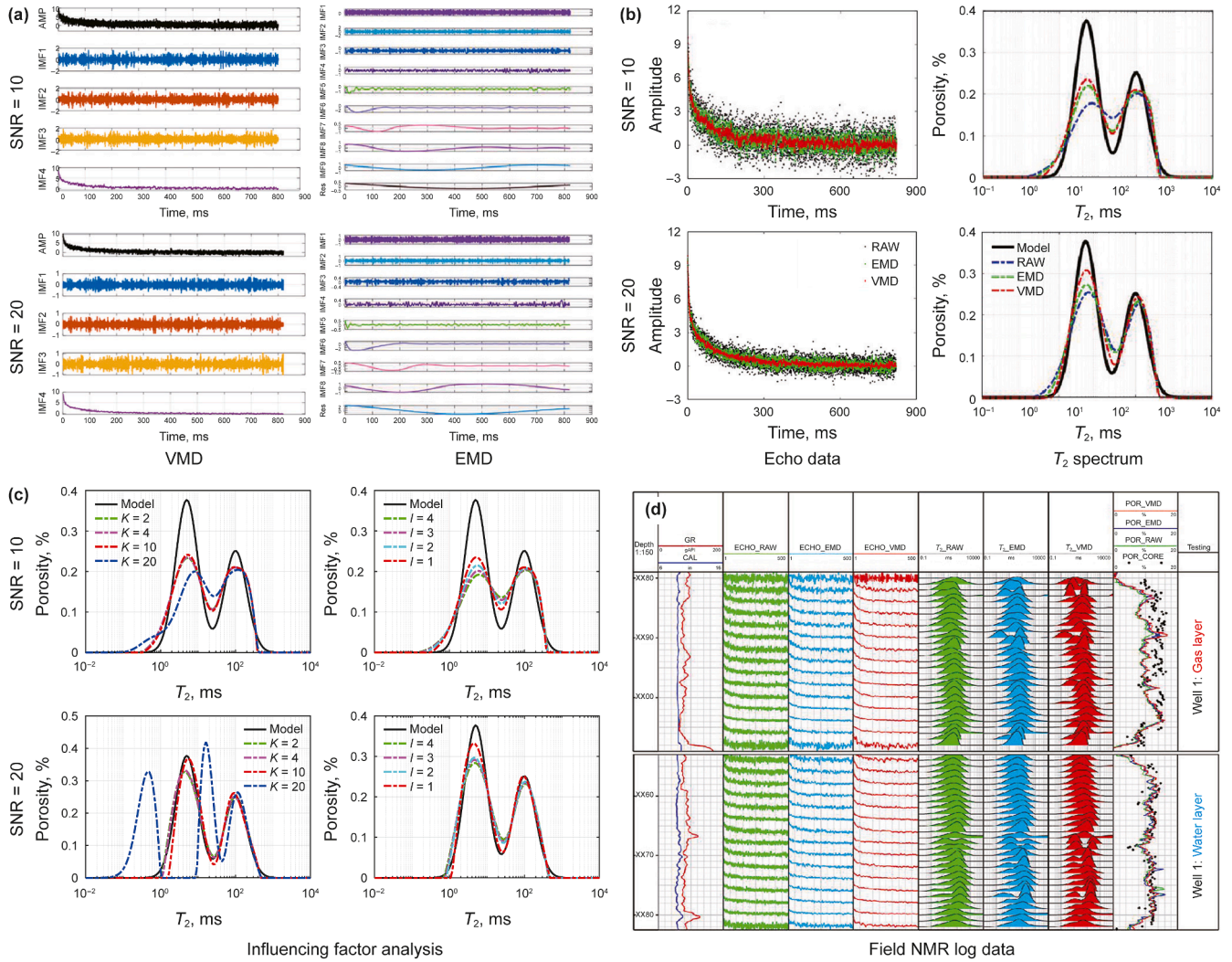
Fig. 5 compares the EMD and VMD methods for denoising echo signals at different SNRs. In the EMD method, high-frequency IMFs were removed and the remaining IMFs combined with the residual to reconstruct the signal. The VMD method selected effective IMFs based on Equation (10). Both methods reduce noise, but VMD-denoised  $T_2$  spectra better match the original model, indicating superior performance. Analysis shows that using an optimal number of IMFs ( $K_{\text{opt}}$ ) improves spectrum fidelity, while over-decomposition or including too many IMFs can introduce spurious peaks and noise, causing peak shifts. In field NMR data, both methods enhance spectral resolution, yet VMD yields clearer bimodal  $T_2$  distributions and porosity estimates closer to core measurements, demonstrating its practical advantage.

In conclusion, the VMD method provides more stable and noise-robust decomposition results than the EMD method, making it particularly advantageous for NMR echo denoising. Nonetheless, its performance remains sensitive to parameter settings, which encourages future efforts toward automatic parameter selection and lightweight hybrid VMD-AI models.

### 3.1.3. Wavelet-based method

The concept of wavelets was introduced early on, but it did not gain widespread attention until the 1980s. In 1981, Mallat proposed the theory of wavelet analysis, which laid the foundation for the significant subsequent development of wavelet theory (Rezaeiifar and Krishnaprasad, 1993). Wavelet analysis was developed from the traditional Fourier analysis methods and has since become widely used (Wu and Huang, 2010).

Fourier-based denoising methods was limited in their ability to represent the localized features of signals and faced significant challenges in handling nonlinear signals, particularly in the early stages of signal processing research. As a result, researchers turned to WT, which offers multi-resolution characteristics and enables the analysis of time-frequency domain information. This multi-resolution capability allows WT to capture specific, localized features of signals, making it highly effective for such analyses (Wood and Johnson, 2015). In denoising theory, WT integrates principles from modern signal processing, enabling the extraction of correlations within the signal based on its inherent properties. By leveraging statistical characteristics, WT achieves more effective signal processing results. Furthermore, continuous advancements



**Fig. 5.** Numerical simulations and field NMR log data processing results for the EMD and VMD methods. (a) Decomposition results of echo data with the VMD and EMD methods; (b) echo data and corresponding  $T_2$  spectra with and without denoising by the EMD and VMD methods; (c) analysis of influencing factor in VMD denoising; (d) field NMR log data processing results using the EMD and VMD methods. Reproduced with permission from IEEE Transactions on Geoscience and Remote Sensing (Guo et al., 2023).

in this field have led to the development of numerous wavelet basis functions, providing greater flexibility and a broader range of choices for practical signal processing applications.

Building upon these general principles, several specific wavelet-based denoising techniques have been established, including the wavelet threshold method, wavelet packet threshold (WPT) method, and wavelet packet domain adaptive filtering (WPDFAF) method, all demonstrating excellent performance in handling noisy signals. Among these, the wavelet threshold method is one of the most classical and widely applied approaches (Meng et al., 2015a, 2015b). Its core principle relies on the multi-resolution analysis property of the wavelet transform. Through wavelet decomposition, the signal is transformed into wavelet coefficients across different scales. Generally, noise energy is primarily concentrated within the detail coefficients of the WT and is relatively uniformly distributed, whereas the coefficients corresponding to the useful signals typically exhibit larger amplitudes.

Wavelet thresholding leverages this distinction by applying an appropriate threshold: wavelet coefficients with amplitudes below the threshold are either discarded or attenuated, while

those above the threshold are retained (Ma et al., 2011). The retained wavelet coefficients are considered to contain the meaningful signal information. Subsequently, an inverse wavelet transform is applied to the processed coefficients to reconstruct a denoised version of the original signal.

The mathematical form of WT can be expressed as:

$$\varphi_{a,b}(\tau) = |a|^{-1/2} \varphi[(\tau - b) / a] \quad (11)$$

where  $a$  is the stretching factor,  $b$  is the translation factor,  $\varphi_{a,b}(\tau)$  is the wavelet basis function obtained by stretching and translating  $\varphi(t)$ .  $\varphi(t) \in L^2(\mathbb{R})$  represents the fundamental wavelet function, which satisfies the following constraint:

$$C_w = \int_{\mathbb{R}} \frac{|\hat{\varphi}(\omega)|^2}{|\omega|} d\omega < \infty \quad (12)$$

For NMR echo data  $M(t)$ , the continuous wavelet transform  $W_M(a,b)$  is the inner product of  $M(t)$  and the wavelet function  $\varphi_{a,b}(\tau)$  (Deng and Xie, 2010a):

$$W_M(a, b) = \langle f, \varphi_{a,b} \rangle = |a|^{-1/2} \int_{-\infty}^{+\infty} M(t) \varphi^*[(t-b)/a] dt \quad (13)$$

where \* denotes complex conjugation.

The reconstructed signal is calculated as:

$$M(t) = \frac{1}{C_w} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{1}{a^2} W_M(a, b) \varphi^*((t-b)/a) da db \quad (14)$$

Effective signal wavelet coefficients typically exhibit larger magnitudes and fewer occurrences, while noise-related coefficients are smaller and more frequent. Coefficients below a specified threshold are set to zero during the thresholding process, while those above are retained for reconstruction. This forms the basis of wavelet thresholding denoising. However, selecting an appropriate threshold is critical. If the threshold is too low, some noise are not removed, degrading the reconstructed signal. On the other hand, if the threshold is too high, the useful signal components may be discarded, leading to the loss of useful information. In practical applications, accurate threshold selection greatly influences denoising performance. Commonly used threshold selection rules are summarized in Table 2.

Xie et al. (2014) compared the wavelet threshold method with two other WT-based method (Wu and Huang, 2010). Fig. 6 presents numerical simulations and field NMR data results. The wavelet threshold denoising method uses heuristic thresholding for signal-noise separation, applying a soft threshold function (Wu and Huang, 2010), defined as:

$$\tilde{W}(j, n) = \begin{cases} \text{sign}(W(j, n))(|W(j, n)| - t_3) & |W(j, n)| \geq t_3 \\ 0 & |W(j, n)| < t_3 \end{cases} \quad (15)$$

where  $\tilde{W}(j, n)$  represents the updated wavelet coefficients after thresholding, and  $t_3$  is the selected threshold.

If the absolute value of a wavelet coefficient is less than the identified heuristic threshold, it is set to zero. In contrast, if the coefficient exceeds the threshold, it is reduced by the threshold value while maintaining its original sign. Among various WT-based denoising methods, the wavelet threshold method is relatively simple and offers the lowest computational complexity.

Fig. 6 shows the results of denoising and inversion using three WT-based denoising methods applied to echo data with an SNR of 20. Among them, the wavelet threshold method exhibits the best performance, as the  $T_2$  spectrum obtained after denoising and inversion is closest to the original synthetic model. In practical well log data processing, the computed porosity values from all three methods show a certain degree of increase and converge toward the core porosity after denoising. Notably, the wavelet threshold method demonstrates the most outstanding performance. The results it produces are closest to the core porosity, providing more accurate data to support the determination of formation porosity and further validating the feasibility of the wavelet threshold method.

The WT method effectively eliminates low-amplitude noise while preserving high-amplitude signals. The soft thresholding function is particularly effective at retaining the key characteristics of NMR signals, achieving satisfactory denoising results. However, high-frequency components often emerge at a low SNR for the wavelet decomposition. Due to the inherent limitations, the wavelet thresholding algorithm cannot further decompose these relatively high-frequency components, leading to reduced decomposition accuracy and ultimately impacting denoising performance.

To overcome these limitations, the WPT was proposed as an advanced extension of the WT (Meng et al., 2015a). Unlike the conventional WT, which decomposes only approximation coefficients, the WPT performs full-band hierarchical decomposition of both approximation and detail components. This enables more refined time–frequency segmentation (Wu et al., 2011). Following decomposition, optimal thresholding is applied to the wavelet packet coefficients, which is then followed by inverse reconstruction. This approach enables the WPT to achieve superior noise suppression and enhances  $T_2$  spectrum fidelity, especially when the noise energy extends across a wide frequency range. In NMR data processing, WPT has been shown to outperform the classical WT in both synthetic and field datasets, improving SNR and producing porosity results closer to core measurements. As shown in Fig. 7, the WPT-denoised echo signals and corresponding  $T_2$  inversion results exhibit enhanced spectral integrity compared with WT.

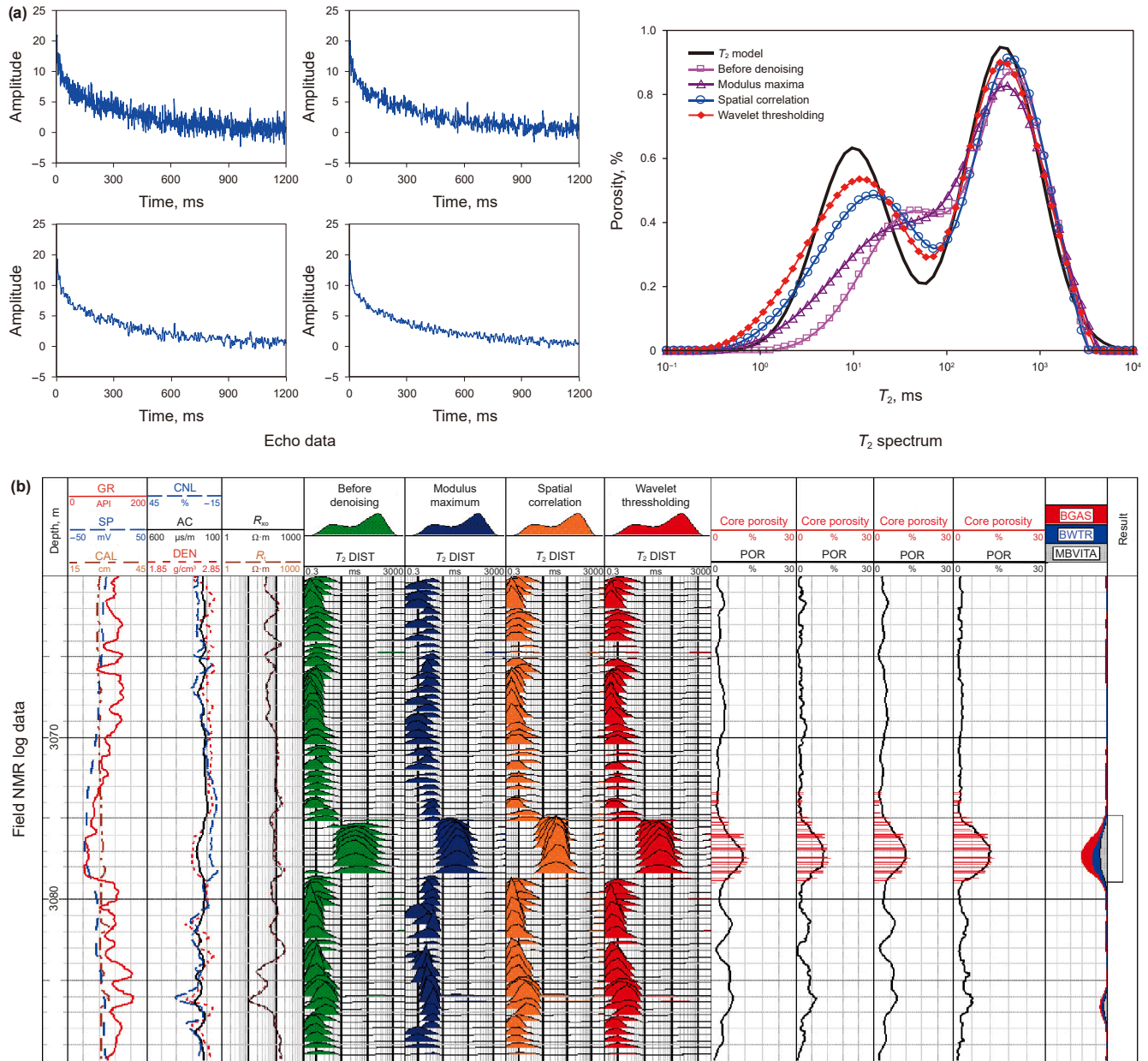
Building on this framework, Meng et al. (2015b) introduced the WPAF method, which integrates WPT with an adaptive least mean squares (LMS) filtering (Chen et al., 2011; Elsayed et al., 2022; Feuer, 1985; Turan and Eleyan, 2016; Wang and Zhao, 2025). Following initial wavelet packet decomposition and thresholding, adaptive filtering is applied to each sub-band. This allows dynamic adjustment of filter weights to more effectively suppress non-stationary noise (Meng et al., 2015b; Zhang and Ni, 2020; Zheng and Zhang, 2007b). This strategy has shown significant advantages under extremely low-SNR conditions, where the traditional WT-based methods may fail to preserve subtle signal features. Notably, the WPAF method provides greater improvements in low-SNR regions compared with the wavelet-domain adaptive filtering. Compared with traditional filtering methods, WPAF offers superior time–frequency localization, enhanced adaptability to non-stationary signals, and greater capability in handling frequency variations. As illustrated in Fig. 8, WPAF-processed echo signals and corresponding  $T_2$  spectra demonstrate enhanced noise suppression and more accurate spectral reconstruction.

Overall, these three wavelet-based denoising methods represent progressive advancements in NMR signal processing. Table 3 summarizes the comparative advantages and limitations of the three methods. The WT provides a simple and efficient baseline for suppressing moderate noise levels. The WPT further refines the decomposition process, achieving improved noise separation and

**Table 2**  
Threshold selection rules for the wavelet thresholding method.

| Threshold type                           | Expression                                                                                                                       | Explanation                                                    |
|------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------|
| Fixed threshold                          | $t_1 = \sigma\sqrt{2 \ln(X)}$ , where $X$ is signal intensity                                                                    | Based on signal intensity, simple formula                      |
| Unbiased likelihood estimation threshold | $t_2 = \sigma\sqrt{w_b}$                                                                                                         | Accounts for the distributional properties of the coefficients |
| Heuristic threshold                      | $t_3 = \begin{cases} t_1 & u \leq \nu \\ \min(t_1, t_2) & u > \nu \end{cases}$ , more complex formula involving $S$ , $K$ , etc. | Combines fixed and likelihood estimation thresholds            |
| Maximum threshold                        | $t_4 = \text{argmin}_\lambda t$                                                                                                  | Optimized for minimum mean square error                        |





**Fig. 6.** Numerical simulations and field NMR log data processing results for the three wavelet threshold-based methods. (a) Echo data and corresponding inverted  $T_2$  spectra with and without denoising; (b) field NMR log data processing results. Reproduced with permission from Journal of Geophysics and Engineering (Xie et al., 2014).

enhanced  $T_2$  spectrum fidelity. Building on this, the WPDFAF introduces adaptive filtering in the transform domain, offering superior robustness at a low SNR.

### 3.1.4. Discrete cosine transformation (DCT)

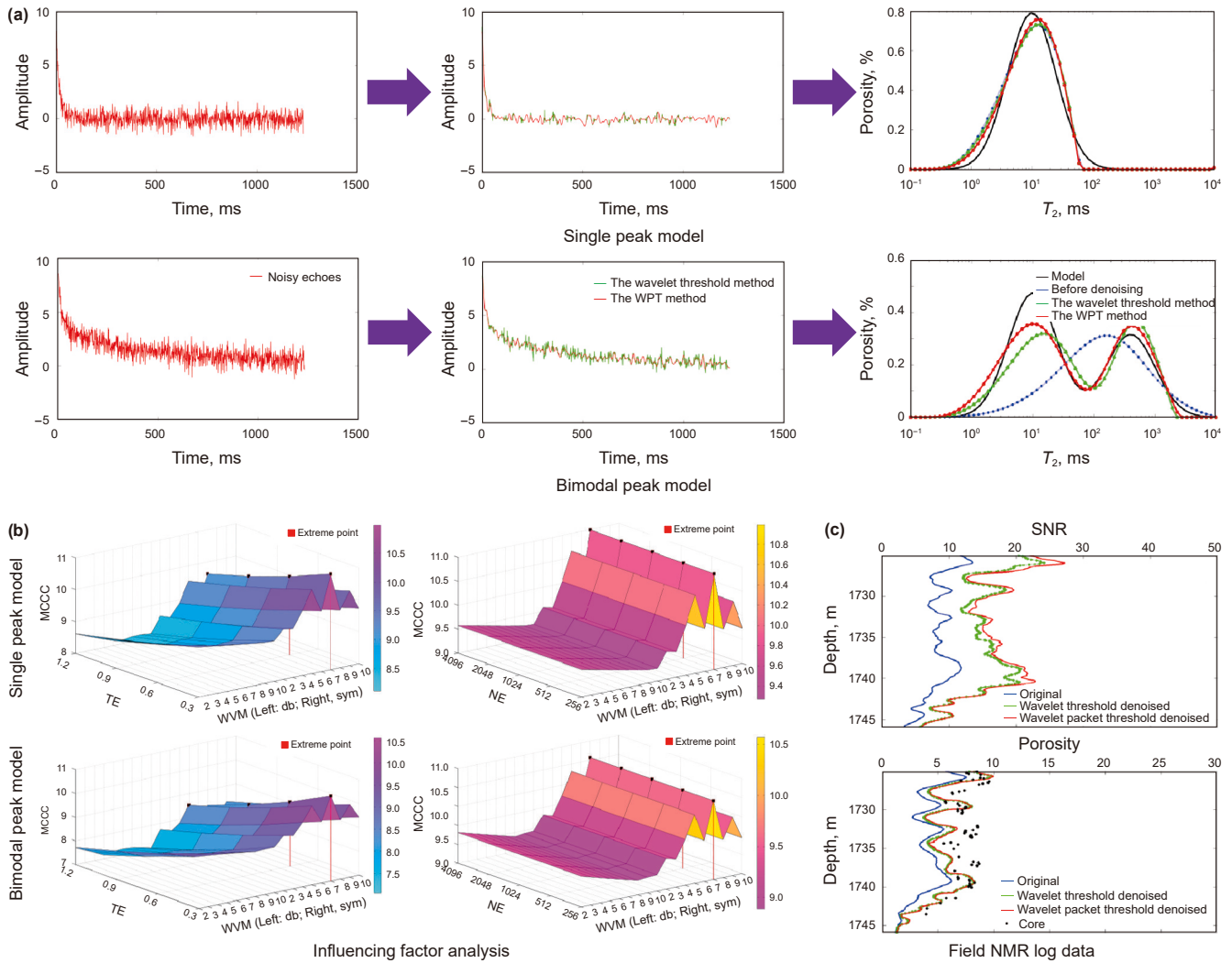
Time-frequency transformation is commonly used for data denoising. Owing to its excellent time-frequency characteristics and multi-resolution properties, WT effectively separates useful signals from noise. Numerous researchers have proposed WT-based denoising methods for NMR echo data. While the early echoes in NMR data contain most of the useful information and are crucial for inversion accuracy, traditional WT-based methods do not specifically target these echoes. To address this gap, Gu et al. (2021c) firstly employed DCT for NMR echo data denoising.

A key advantage of DCT is its energy compaction property, concentrating useful signal information in the low-frequency coefficients, which exhibit higher amplitudes and fewer components. Conversely, noise is typically distributed in the high-frequency coefficients, which have lower amplitudes and more components. Additionally, DCT uses a predetermined transformation kernel, eliminating the need to select decomposition scales (Ahmed et al., 1974).

There are four window-based denoising methods based on DCT, i.e., single window, non-overlapping windows of equal length, overlapping windows of equal length, and variable length windows.

The DCT of the NMR echo data  $M(t)$  is given by:





**Fig. 7.** Numerical simulation and field NMR log data processing results for the WPT method. (a) Echo data and corresponding inverted  $T_2$  spectra with and without denoising; (b) analysis of influencing factors for the WPT method; (c) field NMR log data processing results. db and sym represent Daubechies and Symlet respectively. Reproduced with permission from Applied Magnetic Resonance (Meng et al., 2015a).

$$B(u) = DM(t) \quad (16)$$

where  $D$  represents the DCT coefficients.

After window processing, the retained DCT coefficients must undergo an inverse discrete Cosine transform (IDCT) to reconstruct the denoised signal, expressed as

$$M_i(t) = D'B_i(u) \quad (17)$$

where  $D'$  is the IDCT coefficients.

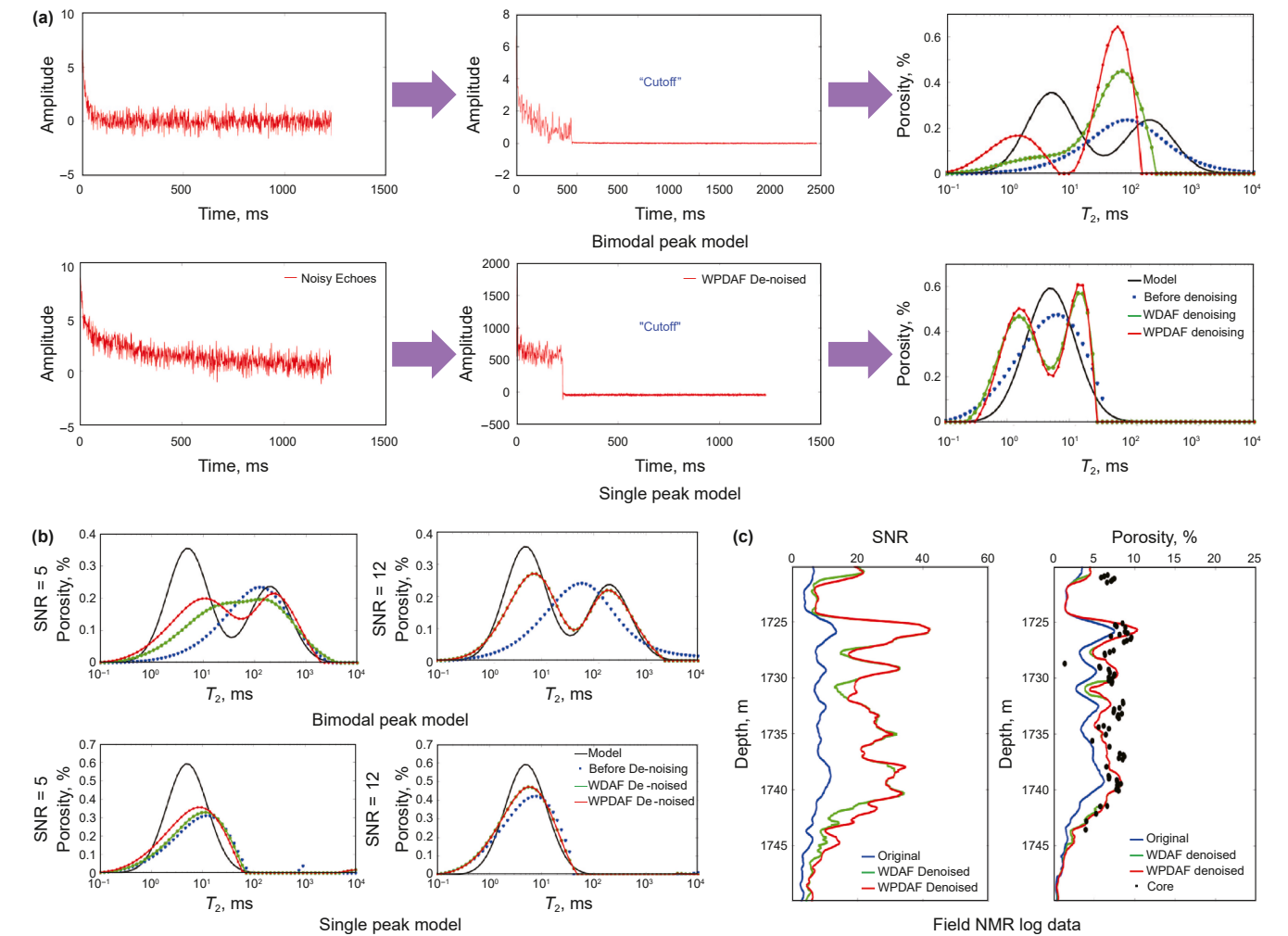
Table 4 presents the denoising process of the four windowing methods based on DCT. Fig. 9 demonstrates the performance of various DCT-based windowing methods in denoising echo data and  $T_2$  spectrum inversion. Among the four methods, the three fixed-window approaches often remove short-relaxation signals, causing a rightward shift in  $T_2$  spectrum peaks and reducing accuracy in identifying short-relaxation components. In contrast, the variable-length window method preserves short-relaxation information more effectively, yielding inverted  $T_2$  spectrum that closely matches the synthetic model. Further 2D denoising results confirm that variable-length windows significantly reduce signal overlap between fluid components, enhancing fluid-type differentiation and improving resolution and interpretation accuracy. In

field NMR log data processing,  $T_2$  spectra obtained using variable-length windows exhibit more pronounced bimodal characteristics and higher inversion accuracy. The calculated porosity values align more closely with core measurements, confirming the method's reliability and practical effectiveness.

In comparison to the fixed-window methods, the variable-length window offers distinct advantages, especially in capturing short-relaxation information and avoiding distortions in  $T_2$  spectrum inversion. However, though the variable-length window improves accuracy, its reliance on dynamic window adjustment can lead to increased computational complexity. Therefore, in practice, the choice for the fixed or variable window should be based on the trade-off between the computational efficiency and the inversion accuracy.

### 3.1.5. Hybrid denoising

The combination of different denoising methods is commonly referred to as hybrid denoising. The primary objective of hybrid denoising is to integrate the advantages of individual techniques to achieve superior denoising performance, providing novel perspectives and strategies for processing NMR echo data (Coifman and Sowa, 2000; Wang et al., 2008).



**Fig. 8.** Numerical simulation and field NMR log data processing results for the WPDFAF method. (a) “Cutoff” phenomenon observed after the echo signal denoising and its effect on the inverted  $T_2$  spectrum; (b) inverted  $T_2$  spectra with and without denoising after eliminating the “cutoff” phenomenon; (c) field NMR log data processing results. Reproduced with permission from Journal of Geophysics and Engineering (Meng et al., 2015b).

**Table 3**  
Advantages and Limitations of the WT, WPT, and WPDFAF denoising methods for NMR echo data.

| Method | Advantages                                        | Limitations                                    |
|--------|---------------------------------------------------|------------------------------------------------|
| WT     | Fast, simple, low computational cost              | Cannot refine high-frequency noise             |
| WPT    | Full-band decomposition, better noise suppression | Parameter-sensitive, higher computational cost |
| WPDFAF | Adaptive noise control, excellent at low SNR      | Risk of “cutoff” artifacts if mismatched       |

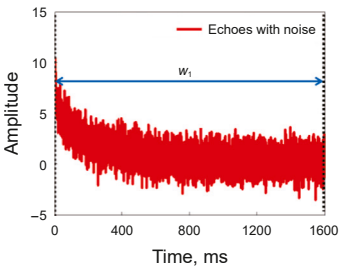
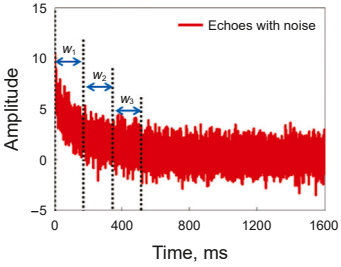
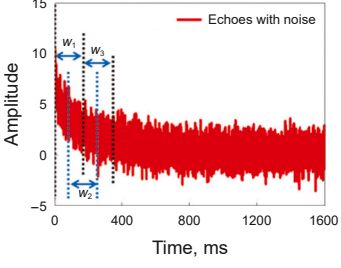
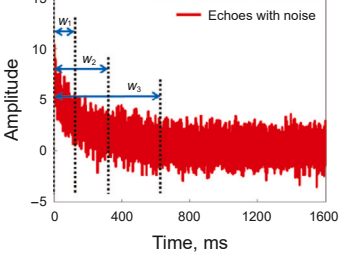
Several factors should be considered when selecting methods for hybrid denoising methods. First, methods based on fundamentally different principles can be combined. For instance, time-frequency analysis-based methods and statistical model-based methods exhibit significantly different response mechanisms to noise and signal components. This difference enables them to capture and process signals and noise from distinct perspectives, leading to more comprehensive noise suppression and better signal preservation in hybrid processing. Second, methods with significant differences in performance characteristics can also be effectively combined (Islam et al., 2017). High-performance methods often exhibit unique advantages in handling specific signal features or noise types; thus, integrating them with other approaches enhances complementarity and overall performance in hybrid denoising. In terms of hybrid strategies, the two main

approaches are cascade hybridization and parallel hybridization. Cascade hybridization follows a sequential strategy, where the output of one denoising method serves as the input for the next, forming a continuous processing pipeline. In contrast, parallel hybridization applies multiple denoising methods simultaneously and integrates their outputs using techniques such as averaging or weighted averaging.

This study has systematically elaborated on various denoising methods based on mathematical transformations. Based on the hybrid denoising concept, any two of these methods can be combined, such as EMD-WT, VMD-SVD, and EMD-SVD. However, the practical effectiveness of these hybrid methods for NMR logging echo data requires validation through comprehensive numerical simulations and comparisons with real well-log data.

**Table 4**

Denoising processes of four windowing methods based on DCT.

| Window type                                        | Processing steps                                                                                                                                                                                                                                                                                                  | Window schematic                                                                      |
|----------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
| Single window (Method 1)                           | <ol style="list-style-type: none"> <li>1. DCT of signal <math>M(t)</math> to obtain <math>B(u)</math></li> <li>2. Retain low-frequency coefficients, set others to zero</li> <li>3. IDCT of retained coefficients to reconstruct <math>M_1(t)</math></li> </ol>                                                   |    |
| Non-overlapping windows of equal length (Method 2) | <ol style="list-style-type: none"> <li>1. DCT of each window <math>M_i(t)</math> to obtain <math>B_i(u)</math></li> <li>2. Retain low-frequency coefficients</li> <li>3. IDCT per window to reconstruct <math>M_{2,i}(t)</math></li> <li>4. Splice <math>M_{2,i}(t)</math> to form <math>M_2(t)</math></li> </ol> |    |
| Overlapping windows of equal length (Method 3)     | <ol style="list-style-type: none"> <li>1. Define windowed data <math>M_i(t)</math></li> <li>2. Apply Method 2 steps</li> <li>3. Weighted average of <math>M_{3,i}(t)</math> to obtain <math>M_3(t)</math></li> </ol>                                                                                              |   |
| Variable length windows (Method 4)                 | <ol style="list-style-type: none"> <li>1. Apply Method 3 steps</li> <li>2. Calculate standard deviation (STD) across windows; set early echoes with STD above threshold to zero</li> <li>3. Weighted average of retained echoes to form <math>M_4(t)</math></li> </ol>                                            |  |

For example, the EMD-SVD method proposed by Zhao et al. (2025) adopted a cascaded hybridization strategy. This method effectively overcomes the limitations of using EMD and SVD individually for NMR echo data denoising while fully leveraging their respective strengths. Specifically, the echo data were first decomposed using the EMD method into low- and high-frequency IMF components and a residual. Next, the SVD method was employed to denoise the high-frequency IMF components. Finally, the low-frequency IMF components, the denoised high-frequency IMF components, and the residual are summed to reconstruct the denoised signal.

The threshold for signal-to-noise separation in EMD-SVD is determined using the optimal singular value hard threshold (OSVHT). This is achieved by calculating the cumulative energy of the singular values from the SVD and selecting a truncation hard threshold that ensures the retained singular values adequately represent the signal while removing those corresponding to the noise.

The optimal truncation hard threshold, denoted as  $\sigma_{ht}$ , is expressed as:

$$\sigma_{ht} = \sum_{i=1}^r 10 \left\lfloor \log \frac{\sigma_{\min}}{\sigma_{\max}} \right\rfloor \sigma_i^2 \quad (18)$$

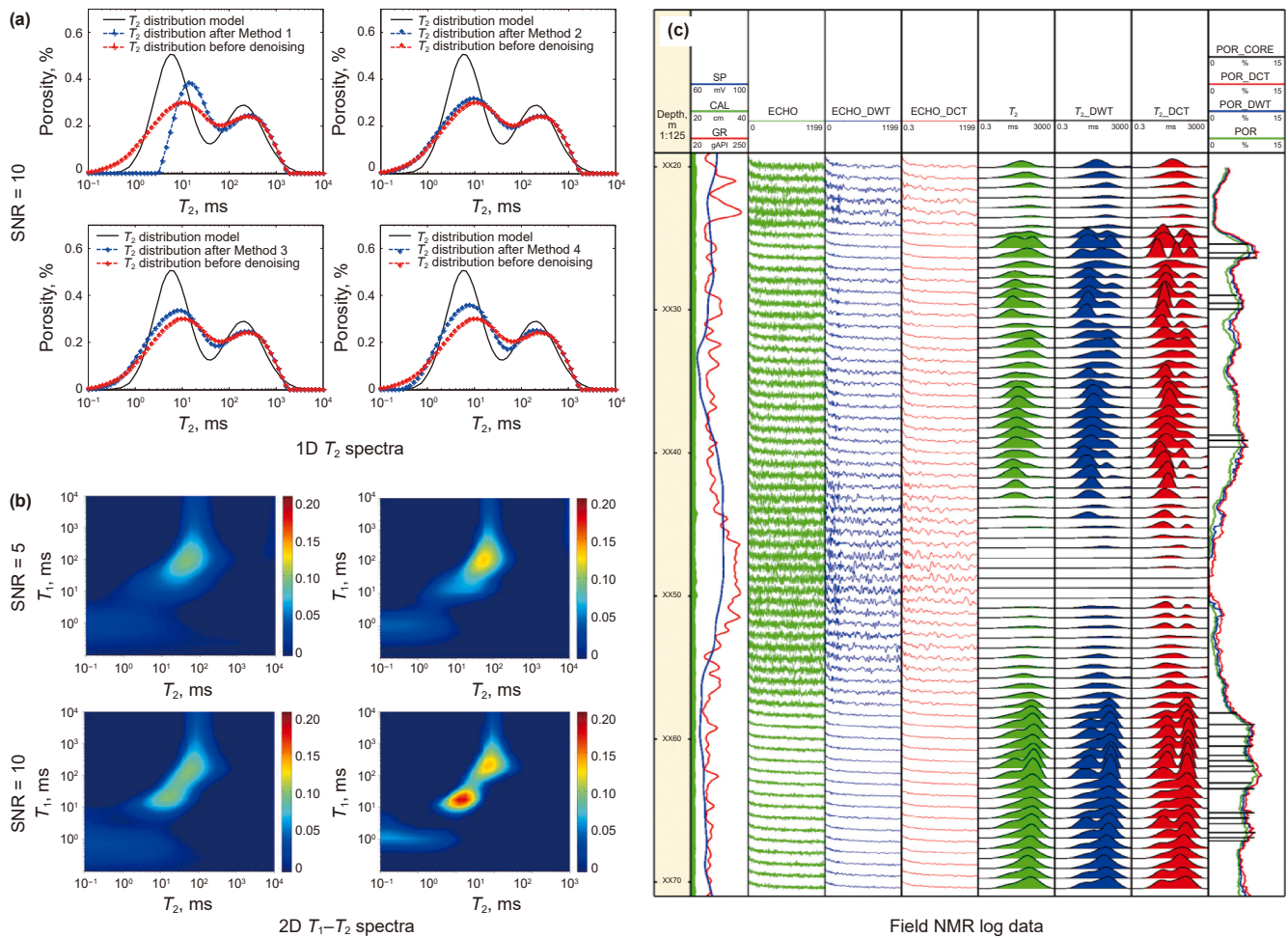
where  $\lfloor \cdot \rfloor$  represents the floor function.

After setting the singular value components representing noise, which are smaller than  $\sigma_{ht}$ , to zero, a new diagonal matrix  $\mathbf{S}'$  is obtained. It is then substituted into Eq. (5) to derive the updated Hankel matrix, from which the denoised signal after EMD-SVD can be reconstructed.

Fig. 10 compares the performance of the EMD, SVD, and EMD-SVD methods in denoising echo data and inverting the  $T_2$  spectrum. While all three methods effectively reduce noise, the EMD-SVD method preserves more useful signal components, resulting in a  $T_2$  spectrum that more closely matches the model. In the inverted 2D  $T_1$ - $T_2$  spectra, EMD-SVD reduces fluid component overlap, allowing for better fluid identification.

For echo data at SNR = 20, selecting the appropriate number of IMFs for SVD processing is critical. When  $j = 4$ , some useful signals





**Fig. 9.** Numerical simulation and field NMR log data processing results for the DCT method. (a) Inverted 1D  $T_2$  spectra with and without denoising; (b) inverted 2D  $T_1$ - $T_2$  spectra after denoising using the variable-length windows; (c) field NMR log data processing results. Reproduced with permission from Journal of Petroleum Science and Engineering (Gu et al., 2021c).

are lost, leading to inaccurate representation of  $T_2 < 3$  ms. However,  $j = 3$  gives a  $T_2$  spectrum more aligned with the model, making it the optimal choice. The SVD method requires data to be structured into a Hankel matrix, with the row number  $C$  chosen as a fraction of the total echoes. For echo data with  $m = 4096$ , setting  $C$  to half the total number of echoes provides the best noise suppression. In field NMR log data, all methods suppressed noise effectively, but the EMD-SVD method yielded clearer bimodal  $T_2$  spectra and porosity estimates closer to core measurements, validating its reliability.

In conclusion, the EMD-SVD method stands out for its ability to effectively denoise NMR echo data while preserving the important signal characteristics. By combining the adaptive decomposition capabilities of EMD with the robust noise suppression of SVD, this method enhances both signal clarity and inversion accuracy. It is particularly advantageous in scenarios where precise identification of fluid components is critical, such as in low-SNR conditions or complex geological formations. However, it is important to carefully select the number of IMFs and the matrix configuration to optimize denoising performance, as inappropriate choices can lead to signal loss or inadequate noise suppression.

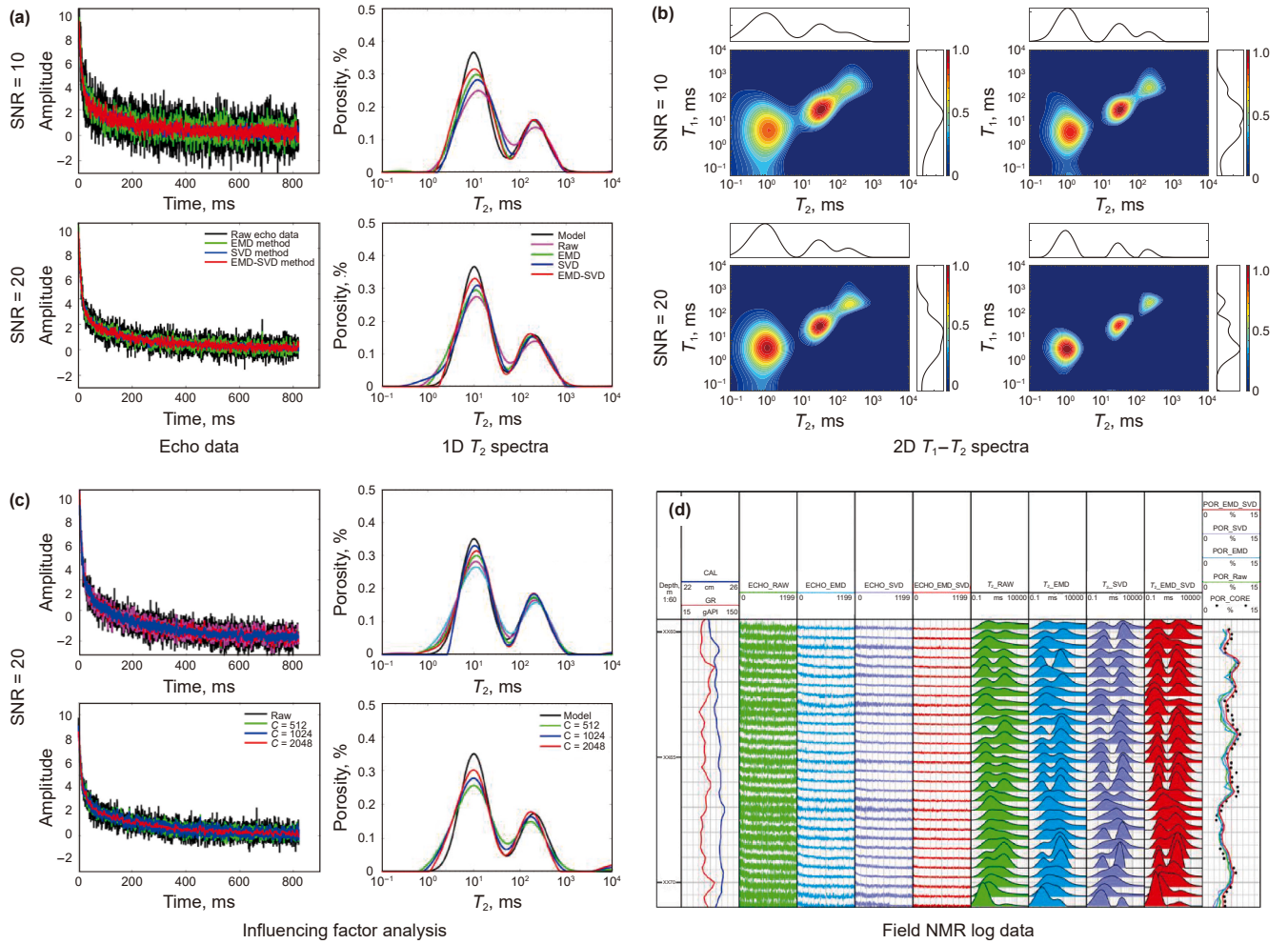
### 3.2. Morphological filtering method

Mathematical morphology is a mathematical framework used to analyze and describe images, geometric shapes, and structures

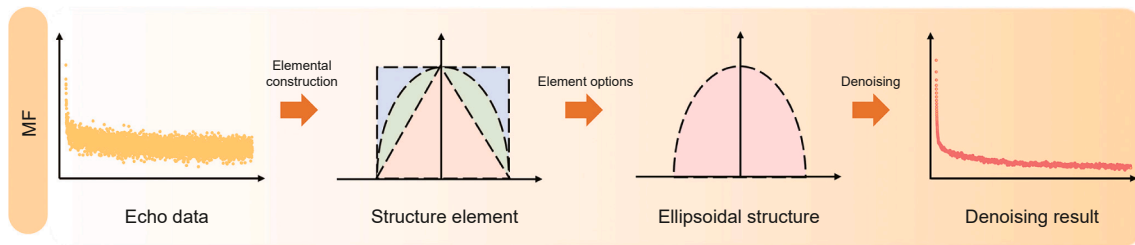
(Gao et al., 2021). Originating in the 1960s, it was initially applied in image processing and later extended to signal processing. This approach enables the identification and quantification of shape features in images while providing a rigorous mathematical basis for their description and analysis (Liu and Fang, 2008).

In mathematical morphology, the fundamental operations commonly employed include erosion and dilation (Cao and Li, 2015; Gao et al., 2021). The erosion operation performs a local transformation by replacing each point with the minimum value within its neighborhood, whereas the dilation operation replaces each point with the maximum value within its neighborhood. In addition to erosion and dilation, mathematical morphology also includes opening and closing operations. The opening operation consists of erosion followed by dilation, while the closing operation involves dilation followed by erosion. By combining and iterating these fundamental operations, mathematical morphology facilitates shape analysis, edge detection, and image enhancement. Depending on the nature of the target data, mathematical morphology can be classified into binary morphology and grayscale morphology (Sukassini and Samy, 2016). Binary morphology is primarily applied to image processing, whereas grayscale morphology is mainly used in signal processing.

Fig. 11 shows a schematic diagram of the morphological filtering method for NMR echo data denoising. The selection of appropriate structuring elements during the denoising process plays a crucial role in effectively suppressing noise interference.



**Fig. 10.** Numerical simulation and field NMR log data processing results for the EMD-SVD method. (a) Echo data and corresponding inverted 1D  $T_2$  spectra with and without denoising by the three methods; (b) inverted 2D  $T_1$ - $T_2$  spectra based on the EMD-SVD method with and without denoising; (c) analysis of influencing factors for the EMD-SVD method; (d) field NMR log data processing results. Reproduced with permission from Magnetic Resonance Letters (Zhao et al., 2025).



**Fig. 11.** Schematic diagram of denoising process with the morphological filtering method. MF, morphological filtering.

The operations of erosion, dilation, opening, and closing on noisy NMR echo data  $M(t)$  are defined as follows:

$$\begin{aligned}
 (M \ominus g)(i \cdot T_E) &= \min\{M((i+k) \cdot T_E) - g(k \cdot T_E)\} \\
 (M \oplus g)(i \cdot T_E) &= \max\{M((i-k) \cdot T_E) + g(k \cdot T_E)\} \\
 (M \odot g)(i \cdot T_E) &= ((M \ominus g) \oplus g)(i \cdot T_E) \\
 (M \oslash g)(i \cdot T_E) &= ((M \oplus g) \ominus g)(i \cdot T_E)
 \end{aligned} \quad (19)$$

where  $M$  represents the echo amplitude at time  $i \cdot T_E$ ,  $g$  denotes the structural element, and  $T_E$  is the echo interval.  $\ominus$  is the erosion operator,  $\oplus$  is the dilation operator,  $\odot$  is the opening operator, and  $\oslash$  is the closing operator.

The filtering performance is quantitatively evaluated using two parameters: the root mean square error ( $RMSE_{inv}$ ) between the  $T_2$  spectrum obtained from the filtered echo data inversion and the  $T_2$  spectrum model, as well as the porosity ( $\phi_{inv}$ ) derived from the  $T_2$  spectrum after inversion of the filtered echo data. The calculation formulas for these two parameters are as follows:

$$RMSE_{inv} = \sqrt{\frac{1}{N} \sum_{i=1}^N (f_{inv,i} - f_{mod,i})^2} \quad (20)$$

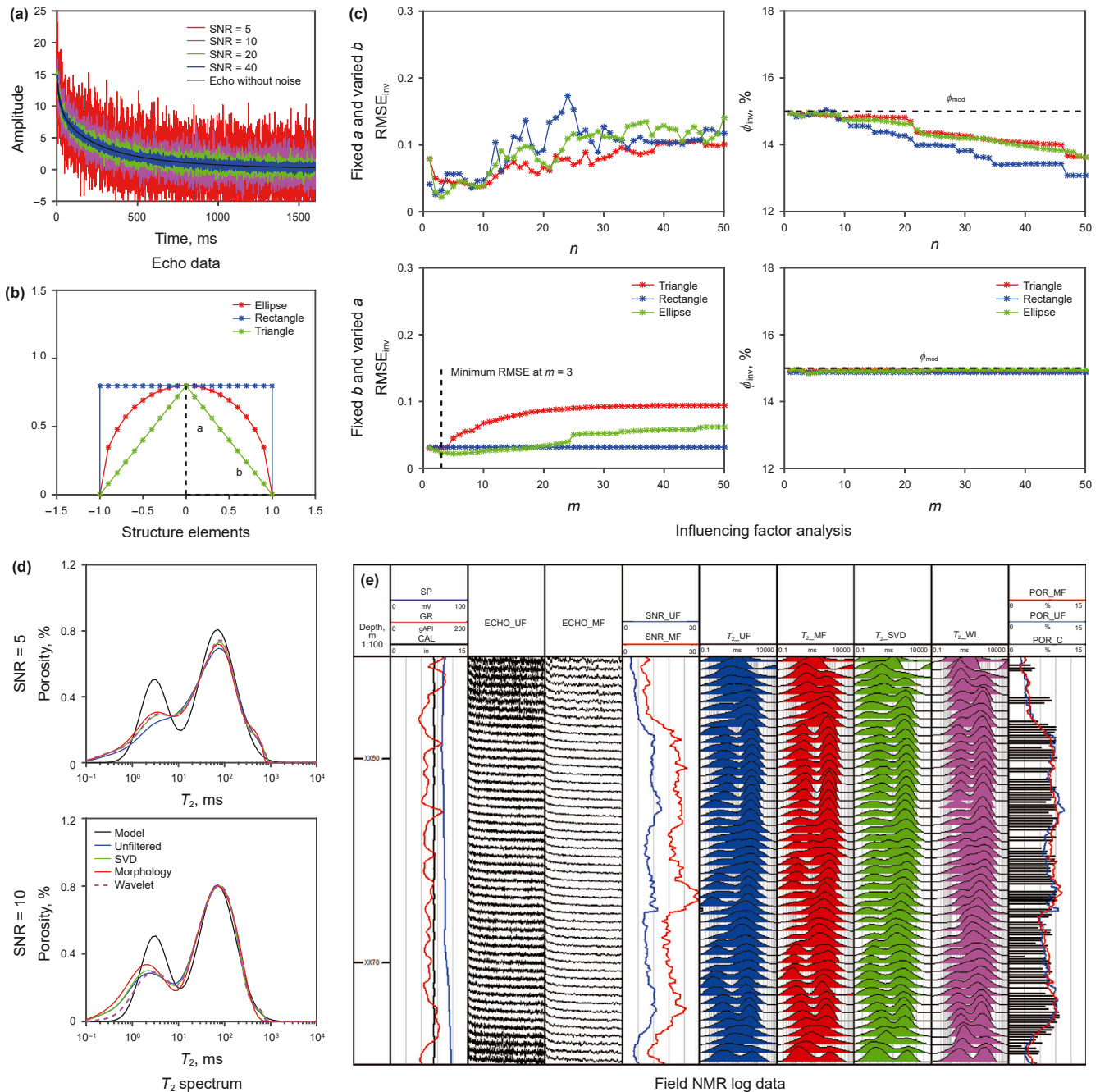
$$\phi_{\text{inv}} = \sum_{i=1}^N f_{\text{inv},i} \quad (21)$$

where  $N$  is the predetermined number of the  $T_2$  distribution.  $f_{\text{mod},i}$  is the amplitude at the  $i$ th  $T_2$  of the model, and  $f_{\text{inv},i}$  is the amplitude at the  $i$ th  $T_2$  of the inversion result.

When applying a morphological filter to echo data, signals from short relaxation components with durations smaller than the width of the filter's structural element may also be removed. Therefore, investigating the impact of the structural element size on filtering performance is crucial. A structural element is defined

by three parameters: height ( $a$ ), width ( $b$ ), and shape. Commonly used structural element shapes include elliptical, triangular, and rectangular forms, as shown in Fig. 12(b).

Gao et al. (2021) analyzed the influence of the structuring element on filtering performance using the simulated echo data at  $\text{SNR} = 20$ . Elliptical, rectangular, and triangular elements were tested by varying the width  $b = n \cdot T_E$  (with height  $a = 5 T_E$ ) for  $n$  ranging from 1 to 50. The  $\text{RMSE}_{\text{inv}}$  curves of all shapes showed oscillations but generally increased with  $n$ , reaching a minimum near  $n = 3$ . The deviation between inverted porosity ( $\phi_{\text{inv}}$ ) and model porosity ( $\phi_{\text{mod}}$ ) also increased with  $n$ , with the rectangular



**Fig. 12.** Numerical simulation and field NMR log data processing results for the MF method. (a) Echo data with and without noise at different SNRs; (b) three types of structuring elements; (c) RMSE and porosity obtained from inverse calculations after filtering with the three structuring elements; (d) inverted  $T_2$  spectra with and without denoising by the MF method; (e) field NMR log data processing results. Reproduced with permission from Geophysics (Gao et al., 2021).



element showing the largest errors. To preserve short relaxation components,  $b$  is typically set between  $T_E$  and  $4T_E$ , where elliptical and triangular elements demonstrating optimal performance. When fixing  $b = 3T_E$  and varying  $a = m \cdot T_E$ , with  $m$  ranging from 1 to 50, the  $RMSE_{inv}$  for the rectangular element remained nearly constant, while that of the triangular and elliptical elements first decreased then increased, reaching minima at  $m = 3$ . Among the tested elements, the elliptical element consistently achieved the lowest  $RMSE_{inv}$  and slightly superior porosity accuracy.

Fig. 12 illustrates the denoising echo data and inverting the  $T_2$  spectrum. A comparison of denoising results for echo data shows that all methods enhance the bimodal structure of the  $T_2$  spectrum. The MF method provides higher accuracy in the short relaxation component. In field NMR log data processing, the MF method significantly improves the SNR. The inverted  $T_2$  spectrum shows clearer bimodal features, higher accuracy, and better resolution. The porosity after MF is closer to the porosity from the core experiments. The MF method is proven to be an effective and stable approach for improving NMR echo data SNR and enhancing  $T_2$  inversion quality. Compared with the other two methods, the MF method offers a simpler algorithm and faster processing speed. However, it may occasionally misprocess signals, leading to artificial details or distortions.

Overall, the gray-scale morphological filtering method effectively suppresses noise and preserves the essential relaxation features of NMR signals. However, its performance is sensitive to the selection of structuring element parameters, and inappropriate configurations may lead to loss of short-relaxation components or incomplete noise removal.

### 3.3. Artificial intelligence method

In recent years, AI-based methods have emerged in the field of NMR echo data denoising, leveraging their powerful data processing and pattern recognition capabilities to provide innovative solutions for enhancing data quality (Dong et al., 2022; Reichstein et al., 2019). The application of AI in NMR echo data denoising primarily involves utilizing machine learning and deep learning techniques to establish novel signal processing frameworks (Altenhof et al., 2023; Ghose et al., 2020; Li et al., 2024). Fig. 13 shows a schematic diagram of the AI-based methods for NMR echo data denoising.

Machine learning algorithms can automatically learn patterns and rules from large datasets, while deep learning enhances this

process through the construction of multi-layered neural networks capable of extracting and representing data features at greater depth (Bloch et al., 1994; Reichstein et al., 2019). The integration of these technologies introduces new possibilities for improving NMR echo data quality.

The core of this approach lies in constructing and training neural network models. Through repeated training on input data, neural networks can deeply learn and capture the intrinsic characteristics of both signals and noise across time, frequency, and other multidimensional domains (Wang et al., 2024). For instance, in the time domain, signals and noise exhibit distinct variation patterns, where neural networks can learn the periodic or continuous nature of signals and the stochastic nature of noise fluctuations. In the frequency domain, differences in frequency distributions allow neural networks to identify the primary frequency ranges of signals and the energy distribution of noise across various frequency bands.

Once adequately trained, the neural network can accurately identify the characteristic patterns of both noise and useful signals in noisy NMR echo data. Through a series of sophisticated computational processes, the network effectively separates noise from the useful signal. Ultimately, the output is a cleaner, higher-quality signal after denoising. This enhanced signal quality significantly improves data usability, laying a solid foundation for subsequent signal analysis and applications.

#### 3.3.1. Dictionary learning

Dictionary learning is a machine learning method designed to learn a dictionary from data that enables sparse representation. In this process, a set of basis vectors (i.e., a dictionary) is identified to efficiently represent the input data as a linear combination of these basis vectors (Luo et al., 2022b). The primary objective of dictionary learning is to obtain an optimal dictionary by minimizing the error associated with the sparse representation of the data.

Specifically, the dictionary learning algorithm iteratively solves for two key matrices: the dictionary matrix  $D$ , whose columns serve as basis vectors, and the sparse coefficient matrix  $\alpha$ , whose rows contain the coefficients that represent each sample in the dictionary space. This framework enables a sparse linear representation of signals through two alternating processes: sparse coding (obtaining coefficients) and dictionary updating (refining basis atoms). Through iterative optimization of these steps, an

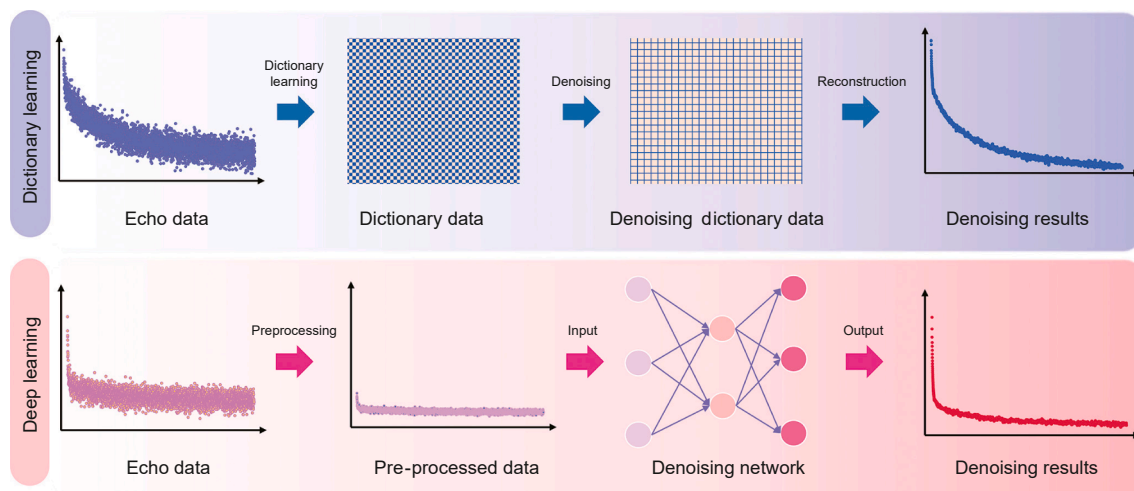


Fig. 13. Schematic diagram of the denoising process with dictionary learning and deep learning methods.

adaptive redundant dictionary and the corresponding sparse coefficients are obtained for the effective signal representation.

The echo data  $M$  is sparsely represented as a constrained optimization problem:

$$\hat{\alpha} \in \operatorname{argmin} \|\alpha\|_0 \text{ s.t. } \|M - D\alpha\|_2^2 \leq \lambda \quad (22)$$

where  $D$  is the dictionary,  $\alpha$  denotes the sparse coefficients, and  $\lambda$  represents the noise threshold.

The sparse representation optimization problem is NP-hard but can be approximated using the orthogonal matching pursuit (OMP) algorithm. The OMP algorithm is straightforward to implement and intuitive in operation. It exhibits high stability and is particularly effective in scenarios with low noise levels.

To fully extract the features of 1D NMR signals, ensure a high-quality redundant dictionary, achieve effective sparse representation, and further reduce the dictionary dimension, [Luo et al. \(2022b\)](#) applied a patching operation to the noisy echo data  $M$ , segmenting it into 1D patches of size  $n \times 1$ . Alternatively, the original data can be structured into a two-dimensional matrix of size  $l \times k$ . A block of size  $n \times m$  (with  $n, m \ll l, k$ ) is selected and scanned across the data matrix, either partially overlapping or covering the largest possible area. Once a large number of patches are obtained, they are transformed into column vectors according to the scanning order and assembled into a training sample set  $S$  (both 1D and 2D patch operations can be used to generate the sample set).

The denoised data  $X$  can be effectively represented by a sparse combination of these patches, leading to the following dictionary update function:

$$\{\hat{a}_{ij}, \hat{X}, \hat{D}\} = \operatorname{argmin}_{a_{ij}, M, D} \lambda \quad (23)$$

$$\|X - M\|_2^2 + \sum_{ij} \mu_{ij} \|\alpha_{ij}\|_0 + \sum_{ij} \|D\alpha_{ij} - R_{ij}X\|_2^2$$

where  $i$  indexes the data blocks and  $j$  indexes the coefficients corresponding to the dictionary basis vectors for each block. Both  $i$  and  $j$  traverse the sparse coefficient matrix.  $X$  is the denoised data, and  $D$  is the dictionary,  $\lambda$  is the regularization factor related to the noise standard deviation  $\sigma$ ,  $\mu_{ij}$  is the residual control factor, and  $R_{ij}$  denotes the block selection operator. The first term  $\lambda \|X - M\|_2^2$  constrains the proximity between the noisy data  $M$  and the denoised data  $X$ . The second term  $\sum_{ij} \mu_{ij} \|\alpha_{ij}\|_0$  ensures continuous optimization of the objective function with respect to the sparse coefficients. The third term  $\sum_{ij} \|D\alpha_{ij} - R_{ij}X\|_2^2$  constrains the proximity between each block  $R_{ij}M$  and its sparse representation  $D\alpha_{ij}$ . The latter two terms together ensure that in the denoised result, the sparse representation of each block  $R_{ij}M$  exists with only finite error.

The sparse dictionary  $D$  is trained using the OMP algorithm to obtain the sparse coefficients  $\alpha_{ij}$  for the local patch data. After denoising each local patch, the overlapping regions from the original segmentation are processed by averaging. Ultimately, the denoised signal is reconstructed as:

$$\hat{X} = \left( \lambda I + \sum_{ij} R_{ij}^T R_{ij} \right)^{-1} \left( \lambda Y + \sum_{ij} R_{ij}^T D \hat{a}_{ij} \right) \quad (24)$$

where  $\lambda$  is the regularization parameter.

[Fig. 14](#) illustrates the denoising and  $T_2$  inversion performance of the dictionary learning method on NMR echo data. A dictionary is first constructed from the echo signals, with block sizes of  $6 \times 6$  and  $7 \times 7$  for SNR = 10 and 20, respectively. These configurations

correspond to dictionary sizes of  $36 \times 144$  and  $49 \times 196$ . The resulting  $T_2$  spectra exhibit a higher resolution, confirming the strong signal representation and denoising capability.

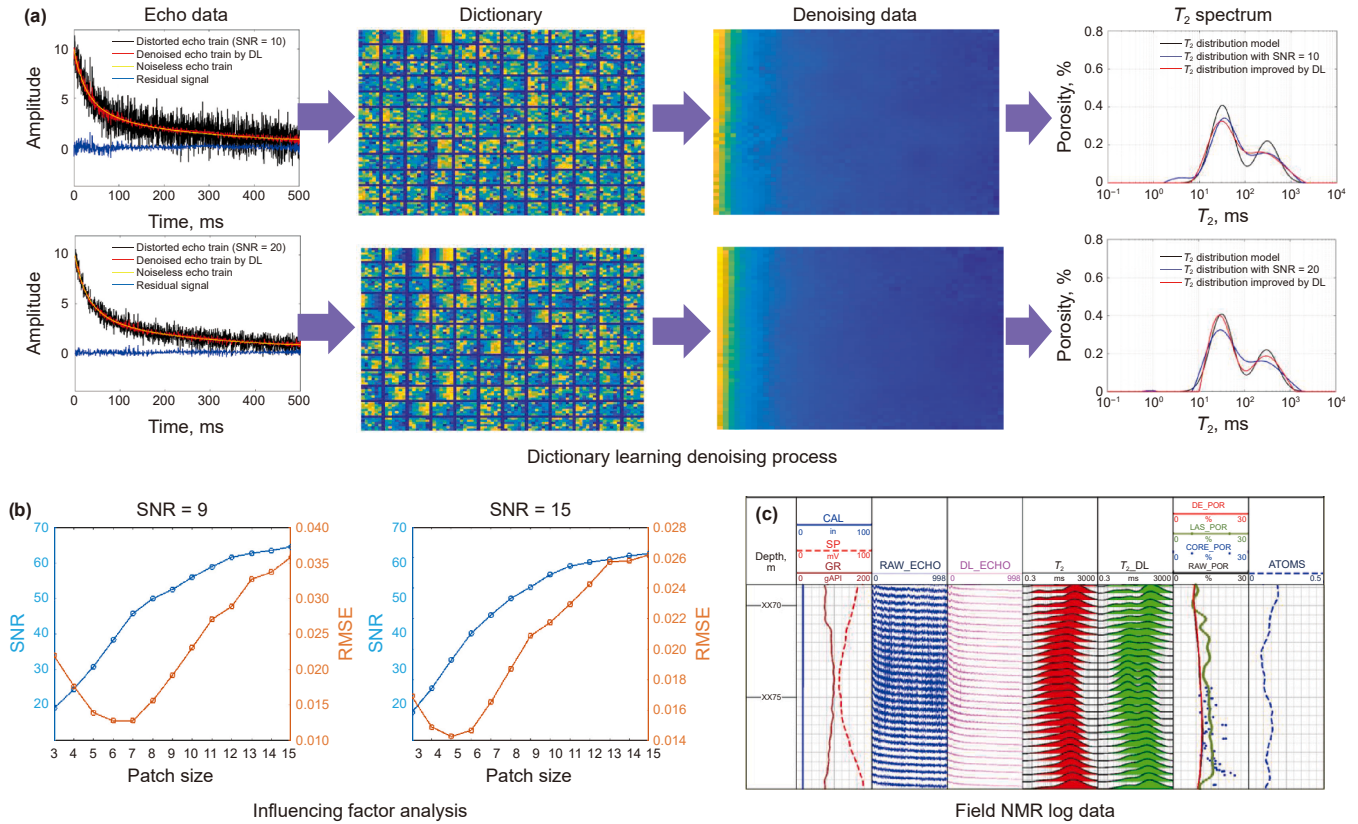
To examine the effect of block size, echo data with SNR = 9 and 15 were processed using various block configurations. Statistical analyses of the denoised SNR and RMSE indicate a nonlinear dependence on block size: larger blocks generally improve SNR but tend to introduce over-smoothing, thereby degrading inversion accuracy. For SNR > 9, a block size  $\leq 6$  is recommended, whereas for SNR < 9, a block size of approximately 7 provides a balanced trade-off between noise suppression and detail preservation. Applications to field NMR log data further demonstrate the method's practical viability and robustness. The dictionary learning approach effectively suppresses noise and improves spectral resolution, yielding clearer bimodal  $T_2$  distributions and porosity estimates that closely align with core measurements. The results also show strong adaptability across varying echo conditions.

Overall, dictionary learning delivers excellent performance in NMR echo data denoising. It preserves fine-scale structural details through sparse modeling and avoids the excessive smoothing commonly observed in linear filtering approaches. The method is particularly well-suited for capturing nonlinear characteristics in echo signals. Nevertheless, several challenges remain, including high computational cost, reliance on iterative optimization, and sensitivity to multiple hyperparameters. In addition, the method requires sufficiently diverse training data; inadequate samples may lead to overfitting or underfitting and consequently impair generalization. Careful hyperparameter tuning and adequate training data are therefore essential for the effective deployment of this method.

### 3.3.2. Deep learning

Deep learning (DL) is a subfield of machine learning that uses multi-layer neural networks composed of interconnected computational nodes (neurons). Each neuron processes input signals through weighted connections and nonlinear activation functions. By iteratively adjusting these weights, neural networks learn task-relevant features and progressively extract higher-level abstract representations. The “depth” in DL refers to the number of layers within the network, which may take the form of architectures such as convolutional neural networks (CNN) and recurrent neural networks (RNN) ([Li et al., 2022b](#)).

DL has achieved remarkable success in domains including computer vision, speech recognition, natural language processing (NLP) ([Zhao et al., 2021](#)), and recommendation systems. Its ability to model complex, nonlinear relationships and automatically learn hierarchical features has led to widespread adoption across scientific and engineering fields. With its rapid development, DL has shown strong potential in the denoising of NMR logging echo data. Traditional denoising methods—such as mathematical transform-based techniques—struggle to handle complex noise patterns and subtle signal characteristics. In contrast, DL models can learn nonlinear mappings directly from data, enabling more effective noise suppression. DL-based denoising typically uses paired noisy and clean signals during training: the network takes the noisy echo as input, applies a trained model for noise removal, and outputs the denoised signal. Some studies have further extended this idea to directly predict the  $T_2$  spectrum using DL models ([Parasram et al., 2021](#)). Through large-scale data-driven optimization, the network parameters are refined to enhance denoising accuracy and generalization. Among various DL architectures, CNNs have become the most widely adopted models in NMR echo processing due to their strong capability in capturing local patterns ([Goodfellow et al., 2014](#)).



**Fig. 14.** Numerical simulation and field NMR log data processing results for the dictionary learning method. (a) Echo data and corresponding inverted  $T_2$  spectra with and without denoising; (b) analysis of influencing factors for dictionary learning; (c) field NMR log data processing results. Reproduced with permission from Petroleum Science (Luo et al., 2022b).

The study of CNNs began in the 1980s, with Time Delay Networks and LeNet-5 representing the earliest CNN architectures. In the 21<sup>st</sup> century, supported by the rise of deep learning theory and advances in numerical computing hardware, CNNs experienced rapid development and became widely used in computer vision and natural language processing (Reichstein et al., 2019). CNNs are feedforward neural networks that employ convolutional operations and hierarchical architectures, enabling automatic feature extraction and translation-invariant representation learning. Owing to this characteristic, they are also referred to as “Shift-Invariant Artificial Neural Networks (SIANN)” (Jifara et al., 2019).

As CNN architectures continued to deepen, researchers observed that simply stacking more layers does not necessarily improve performance. Instead, deep networks often suffer from vanishing or exploding gradients, making them difficult to train effectively. To address these limitations, the residual network (ResNet) architecture was introduced. ResNets incorporate residual blocks with skip connections that allow input features to bypass intermediate layers. This design enables the network to learn residual mappings and mitigates gradient degradation issues. As a result, ResNets can be constructed with hundreds of layers while maintaining stable training dynamics and strong feature extraction capabilities.

Due to their ability to capture fine-grained features and support deep architectures, ResNets have shown distinct advantages in NMR echo denoising tasks. Gao et al. (2023) adopted a ResNet-based framework and proposed the Dn-ResNet architecture for NMR log data denoising. We introduce the details of this method in the following.

Fig. 15 presents the Dn-ResNet architecture and denoising results. Dn-ResNet adopts an encoder-decoder architecture, where the encoder layer captures comprehensive high-level information, while the decoder layer reconstructs noise from the extracted features. After training, Dn-ResNet learns the mapping relationship between noisy echo data and pure noise, enabling the recovery of the useful signals from noisy data.

For a given noise signal dataset  $X = \{M_1, M_2, \dots, M_L\}$  as the network input, where  $L$  is the dataset size, the encoder operates as follows:

$$Z = \sigma(W_e X + b_e) \quad (25)$$

where  $W_e$  represent the weight matrix of the encoder,  $b_e$  the bias vector, and  $\sigma(\cdot)$  the non-linear activation function.  $Z$  represents the feature mapping of the input noisy signal  $M$ . The decoder operates as:

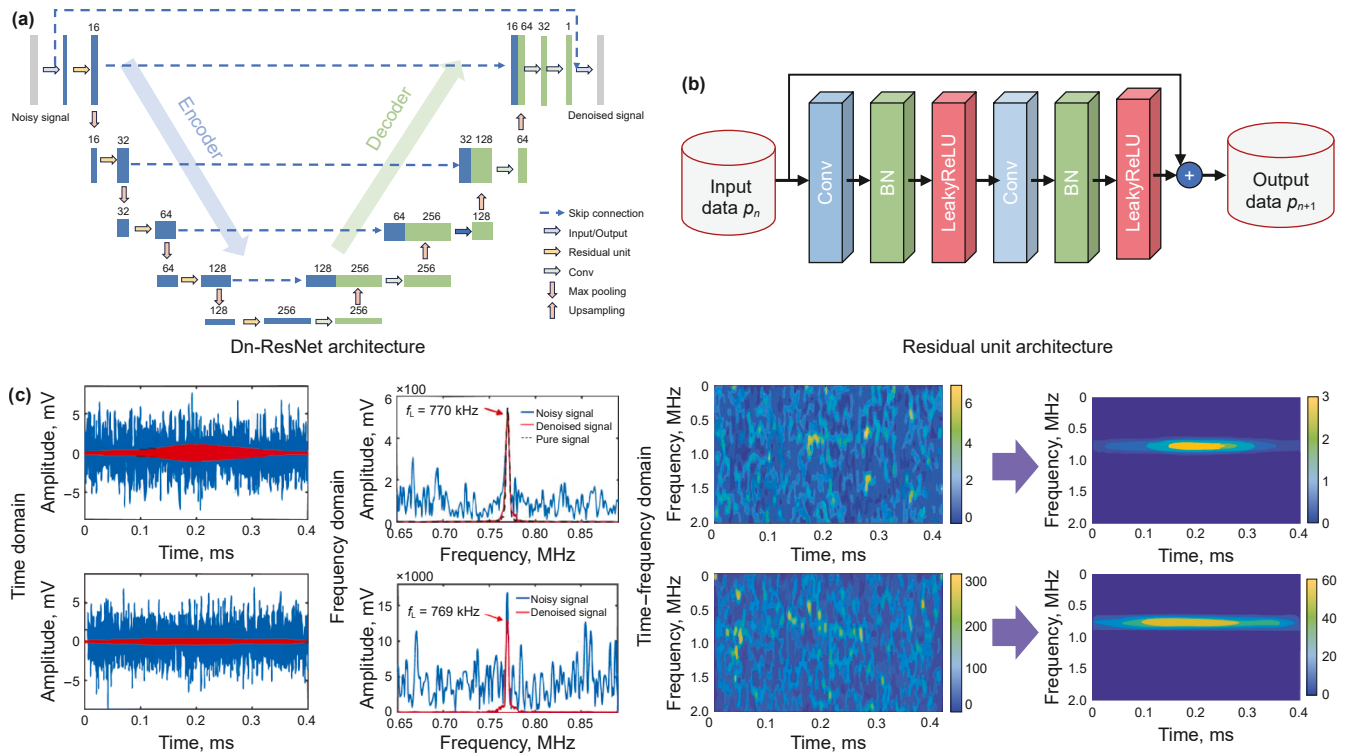
$$N = \sigma(W_d Z + b_d) \quad (26)$$

where  $W_d$  represents the weight matrix of the decoder,  $b_d$  represents the bias vector of the decoder.  $N = \{n_1, n_2, \dots, n_L\}$  represents the reconstructed noise of the feature mapping. Based on the principle of the residual learning aimed at establishing the relationship between noisy signals and pure noise, it can be expressed as:

$$\mathcal{R}(m_i; \Theta) = n_i \approx nn_i \quad (27)$$

where  $\Theta$  represents the trainable parameters within the network.  $m_i$  denotes the  $i$ -th echo signal in the dataset,  $n_i$  denotes the noise





**Fig. 15.** Architectures and numerical simulation results for Dn-ResNet. (a) Architecture of Dn-ResNet; (b) structure of the residual unit; (c) time-frequency denoising results after Dn-ResNet. Reproduced with permission from IEEE Transactions on Instrumentation and Measurement (Gao et al., 2023).

in the reconstruction of this echo signal from the feature map,  $nn_i$  denotes the noise in the desired output of this echo signal.

During the training process, the continual optimization aims to minimize the discrepancy between  $nn_i$  and  $n_i$ . Upon completion of the network training, the denoised outcome can be expressed as:

$$ss_i = M_i - n_i \approx s_i \quad (28)$$

where  $ss_i$  represents the denoised signal,  $s_i$  represents the desired effective signal.

The residual unit is incorporated into the encoder to address degradation caused by increasing network depth. It functions by routing information from input to output through the following residual connections:

$$p_{out} = p_{in} + F(p_{in}) \quad (29)$$

where  $p_{in}$  and  $p_{out}$  represent the input and output of the residual unit, respectively.  $F$  denotes the shortcut connection, transmitting input information through alternating layers of convolution (Conv), batch normalization, and activation functions.

The denoising result of Dn-ResNet demonstrates its effectiveness in suppressing interference noise. The restored signal closely matches the pure signal (Fig. 15). Additionally, the time-frequency representation of the denoised signal shows significant improvement, with previously obscured details becoming clearly visible. By utilizing this model, the number of stacking layers can be reduced while maintaining data quality. This implies that acquisition time can be significantly shortened in practical measurements, thereby improving measurement efficiency.

However, as a supervised machine learning method, the performance of Dn-ResNet remains strongly dependent on large, well-annotated field datasets. Sufficient high-quality labeled data allow the network to learn accurate representations of both signal and noise, thereby enhancing its effectiveness in practical

denoising tasks. In practice, however, field NMR logging data are often limited due to measurement constraints and acquisition costs. This scarcity restricts model performance and underscores the need for effective strategies to mitigate the data bottleneck.

#### 4. Discussion

As summarized in Table 5, the denoising methods reviewed in this study, including the mathematical transformation method, the morphological filtering method, and the AI method, exhibit complementary advantages and limitations in NMR echo processing.

Both wavelet-based methods (WT, WPT, WPAF) and DCT-based methods, provide efficient and interpretable denoising frameworks. They offer strong time-frequency localization and low computational cost, making them suitable for medium-to-high-SNR signals. However, their performance deteriorates in the presence of complex or strongly non-stationary noise.

The mode decomposition methods, including the EMD, SGMD, and VMD method, adaptively decompose signals into intrinsic components without relying on predefined bases. Among them, The VMD method achieves superior mode separation and stability, whereas the SGMD method effectively suppresses mode mixing but incurs higher computational cost. Although these methods preserve physical interpretability, they remain sensitive to parameter selection and may suffer from over-decomposition under high-SNR conditions.

The hybrid method, such as the EMD-SVD method, leverage the complementary strengths of decomposition and matrix factorization to enhance noise suppression and feature preservation. Nevertheless, determining the optimal number of intrinsic mode functions and appropriate matrix dimensions remains challenging.

**Table 5**

Denoising processes of four windowing schemes based on DCT.

| Method category                    | Representative methods            | Advantages                                                                                           | Limitations                                                                                                     |
|------------------------------------|-----------------------------------|------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------|
| Mathematical transformation method | SVD                               | Efficient noise–signal separation; simple structure; low parameter sensitivity                       | Depends on matrix dimension; may remove weak signal components                                                  |
|                                    | Mode decomposition (EMD/SGMD/VMD) | Adaptive to nonlinear signals; adaptive decomposition; strong physical interpretability              | Mode mixing (EMD); high computational cost (SGMD); sensitive parameter tuning (VMD)                             |
|                                    | Wavelet-based (WT/WPT/WPDAF)      | Adaptive to nonlinear signals; strong physical interpretability; good noise suppression              | Parameter selection critical; computationally more expensive (WPT/WPDAF); may introduce artifacts under low SNR |
|                                    | DCT                               | Concentrates signal energy in low-frequency coefficients; simple implementation                      | Fixed basis reduces adaptability; loss of short-relaxation information                                          |
|                                    | Hybrid (e.g., EMD–SVD)            | Integrates multiple strengths; improved denoising and feature preservation; higher accuracy          | Parameter-dependent; higher computational burden                                                                |
| Morphological filtering method     | Gray-scale morphology             | Preserves edge and structural details; effective for short-relaxation signals; simple implementation | Sensitive to structuring element; potential over-smoothing                                                      |
| AI method                          | Dictionary learning               | Adaptive sparse representation; strong reconstruction capability; flexible feature extraction        | Computationally expensive; patch-size dependent; parameter tuning required                                      |
|                                    | Deep learning                     | End-to-end learning; excellent low-SNR performance; strong generalization                            | Requires large datasets; high training cost; limited interpretability                                           |

The morphological filtering methods, which employ gray-scale structuring elements, excel at preserving key structural features and short-relaxation components. Their performance, however, depends strongly on the shape and scale of the selected structuring elements; inappropriate parameter choices may lead to the loss of fine spectral information.

The AI method, particularly dictionary learning, demonstrate strong adaptability and data-driven feature extraction capability. They produce high-resolution  $T_2$  spectra and porosity estimates consistent with core measurements. Their performance, however, is influenced by block size, dictionary redundancy, and noise characteristics. The deep learning denoising further extends this paradigm by enabling nonlinear mapping and end-to-end feature learning. These models achieve high denoising accuracy and generalization but require large training datasets and substantial computational resources.

## 5. Conclusions and future prospects

In summary, no single method offers universal optimality. The transform- and morphology-based methods are generally more suitable for the medium-to high-SNR conditions, whereas the adaptive decomposition, hybrid, and AI-based methods demonstrate greater robustness under low-SNR or geologically complex scenarios. Recent advances in artificial intelligence have enabled end-to-end denoising frameworks that automatically learn hierarchical and nonlinear representations, marking a significant shift from the traditional model-driven strategy.

Despite these advances, several challenges remain for the AI-based NMR denoising. The limited availability of labeled NMR datasets, model interpretability, and the trade-off between computational efficiency and denoising accuracy continue to constrain practical deployment. Moreover, models trained on data from a specific oilfield or formation may not generalize well to regions with different geological conditions. Given the advantages and limitations of each method, future research should be focused on conducting comprehensive and systematic analyses to develop the targeted optimization strategy for each denoising approach. To meet the increasing complexity of oil and gas exploration and development, NMR log data denoising methods are expected to evolve towards integrated and efficient solutions.

On one hand, integrating multiple methods warrants further investigation, for example, combining the mathematical transformation method with the AI method to leverage their respective strengths. On the other hand, with ongoing advancements in software and hardware technology, the development of light-weight AI models will be a key direction for improving NMR data quality. Such models can perform effectively on low-resource devices while extracting more data features, offering a practical solution for future field applications. In addition, AI-based methods face challenges when applied to field data from diverse geological formations. Models trained on one oilfield may not generalize well to others due to variations in lithology, fluid properties, or logging conditions. To address these limitations, techniques such as transfer learning, domain adaptation, and data synthesis using generative models could be explored. Incorporating physics-informed priors or calibration with local field measurements may further enhance the robustness and practical applicability of AI-based denoising methods.

## CRedit authorship contribution statement

**Jiang-Feng Guo:** Writing – review & editing, Validation, Supervision, Methodology, Funding acquisition, Conceptualization. **Yong-Jie Zhao:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Data curation, Conceptualization, Investigation. **Ran-Hong Xie:** Validation, Supervision, Methodology, Conceptualization. **Li-Zhi Xiao:** Validation, Supervision, Methodology, Conceptualization. **Zhen-Hua Rui:** Supervision, Methodology, Conceptualization. **Si-Hui Luo:** Visualization, Validation, Methodology, Conceptualization. **Guo-Wen Jin:** Validation, Supervision, Investigation. **Pei-Yuan Yan:** Methodology, Investigation, Formal analysis. **Dan Xiao:** Visualization, Validation, Supervision, Methodology.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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