

Review Paper

A critical review of analytical oil flow rate models for vapour extraction and its non-condensable gas-based hybrids

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ABSTRACT

Analytical models are the preliminary reservoir engineering processes performance prediction tools that are quick and less costly compared with experiments and simulations when sufficiently modelled which motivates evaluation of oil drainage rate modelling in the mentioned subjects. The models were derived based on the principles of mass balance (including diffusion/dispersion), Darcy's law, and heat of conduction with minimum convective heat transfer analysis. The majority of the models were based on steady-state flow conditions with a scarcity of models considering unsteady-state conditions. The inverted triangle (linear) and the curved geometries defined the vapour chamber shapes with a lack of curved studies in the hybrids of warm vapour extraction (VAPEX) and non-condensable gas (NCG) with steam. The lateral chamber spreading stage was common in the creation of each model. Limited studies in rising and declining chamber stages were noted especially in the warm VAPEX and hybrids of warm VAPEX and non-condensable gases with steam, enhancing more comprehensive modelling studies that distinguish stages in future works. Hybrids of warm VAPEX and NCG with steam models were discovered to lack validations. A paucity of model uncertainty parameters tuning to match experiments, simulations, and field studies was revealed promoting parameter-tuning sensitivity analysis to estimate most components of uncertainty in the equations and further driving mechanisms exploration in the subsequent research. Modelling modifications are also suggested including sensible heat consideration in the steam heat balance equations that utilized latent heat only.

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1. Introduction

Exploration of heavy oil and bitumen is currently getting attention as a result of the declining trend of conventional oil reserves and the rising fossil fuel consumption (Ado, 2020; Wu et al., 2020; Liu et al., 2022; Kavuba et al., 2026; Zhu et al., 2026). The global reserves of unconventional petroleum, such as bitumen, heavy oil, and extra-heavy oil, exceed six trillion barrels, which is almost three times the proportion of reserves of conventional oil and gas (Pourabdollah et al., 2010; Fayazi and Kantzas, 2019; Wu et al., 2021; Yang et al., 2024) with China, Venezuela, Canada,

and the United States as the top countries for the heavy oil production and exploration, in ascending order.

Steam or in situ combustion technologies are the traditional methods of recovering these unconventional reserves. The common steam methods are cyclic steam stimulation (CSS), steam flooding (SF) and steam assisted gravity drainage (SAGD) (Dong et al., 2019; Yang et al., 2021; Liu et al., 2023). Solvent and non-condensable gases have been proposed to be applied in the heavy oil recovery due to several environmental, technical and economic challenges encountered while using steam as the main injection fluid (Wu et al., 2018; Andriianova and Leung, 2021; Mayo-Molina et al., 2022). Environmentally the concern about significant emissions of greenhouse gases after using natural gas to produce steam has been reported, and technically the thin formations, less than 200 m, deep formations, and reservoirs with the

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surrounding aquifers affect the performance efficiency of the methods (Huang and Babadagli, 2020; Salas Santa et al., 2020). However, during operation, there is significant steam overriding and channelling at the later stage of steam usage which decelerates the recovery and over exceeds the economic utilization of steam. Economically steam usage will be minimized by the use of the mixture of steam and solvents/non-condensable gas (NCG) hence less power is required for steam generation and technically process improvement will be attained (Lawal, 2014; Zhang et al., 2018).

As one of the predictive tools, analytical models in petroleum engineering serve as preliminary technical prediction performance of the reservoir to have the right decisions on the project's implementation before experiments and simulations subjects provide a mechanism for speedy performance forecasting and field-scale optimization, eliminating the need for extremely time-consuming thermal numerical simulations at a moment (Chai et al., 2022; Wu et al., 2022; Chen et al., 2025). Therefore, the subject of the analytical modelling is very important in the reservoir performance evaluation to have several quick recovery enhancement analyses for the future production trend and its impact on profitability.

Vapour extraction (VAPEX) uses solvents commonly propane and butane instead of steam at reservoir temperature to extract higher molecular weight asphaltenes and insoluble particles from heavy oil or bitumen, hence lowering the viscosity of the mixture by the solubilization of heavy oil through mass transfer between solvent and heavy oil/bitumen (Bayestehparvin et al., 2019, 2021). The principles of mass balance and momentum balance across the control element of the solvent vapour-bitumen interface under steady-state flow circumstances were used based on the concepts of diffusion (Fick's mass transfer theories) and Darcy's law to derive the models. The geometries of the models include the linear ones developed by Butler and Mokrys (1989), and Cheperli et al. (2022). The parabolic model by Ma et al. (2017) and the produced linear models by Moghadam et al. (2009) were under the laterally spreading stage, which encourages more research on the remaining stages (rising and declining) while the exponential model by Cheperli et al. (2022) is based on spreading and declining stages, but the circular model by Lin et al. (2014) defines all three stages.

The main disadvantage of the VAPEX process is its slow oil production rate, which results from the mass transfer that is fundamentally slower than heat transmission by conduction and convection in SAGD (Pang et al., 2019). Warm VAPEX, on the other hand, can greatly enhance the rate of oil production since it combines heating (above the saturation temperature of solvent) and solvent dilution effects (Rezaei and Chatzis, 2007). The developed linear geometry model by Motahhari and Khaledi (2018) comprises the boundary layers for heat transport and mass transfer. The mass boundary layer is enclosed by the heat transfer boundary layer. Diffusion and dispersive mechanisms are used in the mass transfer mechanisms resulting in the steady state distribution of the solvent concentration from the interface to the dead heavy oil side and conduction is the heat transfer mechanism that is used to produce a temperature profile which is steady beyond the solvent-bitumen interface. The lack of developed models using the warm VAPEX framework was recognized. However, there are few recognized models in the literature, therefore, more modelling research is required in the future to accommodate more linear geometries models based on the analysis of the drive mechanisms and further on curved geometries features including circular, parabolic, and exponential to have variable options of warm VAPEX process figuring.

In the hybrid warm VAPEX and steam injection process, steam has a major role in the process of heat transmission into the heavy oil/bitumen, while the solvent plays a role mainly in mass transfer (Zargar and Farouq Ali, 2021). Linear geometry served as the foundation for all of the generated models of the solvent/steam-heavy oil/bitumen interface which include the ones by Faradonbeh et al. (2012) and Rabiei Faradonbeh et al. (2014). The principle of the heat of conduction, mass balance and momentum balance were used for the formulation of the models with the addition principle of the constant rate of steam heat injection which need further analysis in the heat balance equation. Faradonbeh et al. (2012) utilized the distribution of temperature under steady-state and distribution of concentration by unsteady state conditions ahead of the steam/solvent interface to the dead heavy oil/bitumen side while Rabiei Faradonbeh et al. (2014) utilized unsteady state conditions for both temperature and concentration profiles. However, a lack of model verification was noted. All of the models under hybrid warm VAPEX and steam injection were created under the laterally spreading stage of the solvent-steam vapour chamber growth, which emphasizes additional research on the geometries, such as extended linear, circular, parabolic, and exponential ones under the laterally spreading chamber growth stage, as well as additional of rising and declining stages to stimulate the modelling works analysis.

The non-condensable gas (NCG) system was revealed to have few developed model with the option of diffusion and non-diffusion processes based on the rate of the mole fraction change of the NCG ahead of the vapour/NCG interface and the dead heavy oil/bitumen, which increases the need for additional research in the future to fully comprehend analyses involving how analytically the NCGs act as the heat insulator from steam heat transfer to the overburdened rocks and steam temperature decrease impacts combined with the steam chamber pressure maintenance as the main three driving mechanisms stimulating heavy oil recovery (Wang et al., 2016; Yuan et al., 2018).

This review article, therefore, tackles the limited development of curved vapour chamber shape modelling in systems of warm VAPEX, hybrid s of warm VAPEX and NCG with steam injection with only revealed linear (invited triangle) vapour chamber shape modelling in these systems. Moreover, the review offers a novel perspective on the role of NCG, highlighting their impact on heat transfer dynamics and steam chamber pressure maintenance. By providing a more comprehensive treatment of these elements, this paper offers solutions to long-standing issues in analytical modelling, ultimately providing a refined and validated approach to optimize the heavy oil recovery processes. Through its detailed analysis and integration of previously overlooked factors, this review presents an improved modelling framework that accounts for the full complexity of heavy oil recovery.

2. VAPEX, warm VAPEX and hybrid warm VAPEX with steam injection

2.1. VAPEX recovery mechanisms and analytical models evaluations

2.1.1. VAPEX recovery mechanisms

The VAPEX process primarily operates by making heavy oil or bitumen less viscous through the removal of the higher molecular weight insoluble solids and asphaltenes from the oil or bitumen by the use of solvents (Bayestehparvin et al., 2019). Additionally, this process may lead to a reduction in interfacial tension and cause oil swelling. Commonly used solvents in VAPEX include propane (C_3H_8), butane (C_4H_{10}), methane (CH_4), hexane (C_6H_{14}), or mixtures of these solvents. Recently, dimethyl ether has emerged as a

promising solvent for warm VAPEX, offering the advantage of being water-soluble, unlike traditional solvents which are water-insoluble (Bakhsh et al., 2024). Gravity forces oil from the reservoir's higher regions up to the producing well at the bottom after mass transfer, particularly through the diffusion and dispersion of molecules of solvent into the heavy oil. The first involves the solvent's dispersion/diffusion into the heavy oil reducing its viscosity and allowing gravity to control capillary forces. The second one involves the gravity-driven transport of the oil which is less viscous to the producer well (Pourabdollah and Mokhtari, 2013). Viscosity drops when solvent molecules permeate into bitumen or heavy oil, and the diluted, lower-viscosity oil flows down to the production well due to gravity.

The gas solvent's diffusion across the acute interface triggered during the mixing of the two fluids by the vapour solvent molecules randomly entering the oil phase at constant volume circumstances can be described by Fick's diffusion Eq. (1). However, simulations and tests/experiments show that diffusion is not the only phenomenon to take into consideration. Convective dispersivity can also play a role in determining the ultimate solvent diffusivity coefficient in heavy oil or bitumen bulk volume.

$$\frac{dG}{dt} = -D_o A \frac{\partial c}{\partial x} \quad (1)$$

where G is the amount of substance spreading/diffusing over a plane; t is the duration, s; D_o is the coefficient of molecular diffusion, cm^2/s ; A is the diffusion cross-sectional area, cm^2 ; c is the volume proportion of the solvent/concentration and x is distance, cm.

2.1.2. VAPEX linear developed models

The first model was generated by Butler and Mokrys (1989) using the principles of Fick's mass transfer/diffusion law and Darcy's laws to establish the mass balance and momentum balance across the control element between the solvent vapour interface and dead heavy oil/bitumen with two physical assumptions below. The resulting analytical balance after the analysis is expressed in Eq. (2).

- (1) The solvent transfer procedure from the chamber of solvent vapour to the volume of bulk the heavy oil/bitumen is steady state.
- (2) The interface of the vapour solvent and bitumen moves at a constant velocity in x -direction as the one explained in Butler (1985).

$$q = 2L\sqrt{2kg\Phi\Delta S_o N_s h} \quad (2)$$

where L is the length of the production well, cm; k is the average permeability of the zone, cm^2 ; g is the gravity acceleration, cm/s^2 ; Φ is the porosity of the reservoir, fraction; ΔS_o is the change of oil saturation in the swept area; h is the effective drainage height from the reservoir top to the production well, m; N_s denotes the dimensionless group number.

$$N_s = \int_{C_{\min}}^{C_{\max}} \frac{\Delta\rho D_s (1 - C_s)}{\mu C_s} dC_s \quad (3)$$

where $\Delta\rho$ is the difference in density between live oil (diluted) and pure solvent in g/cm^3 ; D_s is the solvent molecular diffusivity, cm^2/s ; C_s is the fraction of volume of the solvent, fraction; μ is the viscosity of the diluted oil with solvent concentration C_s , $\text{mPa}\cdot\text{s}$.

Butler and Mokrys (1989) carried out several tests utilizing bitumen from Athabasca and Suncor and toluene as a solvent. The recovery rates from their experiments fit within the predicted

range of their analytical model. Dunn et al. (1989) investigated the recovery rates of Athabasca bitumen in porous sand pack media by introducing gaseous solvents using the same mathematical model created by Butler and Mokrys (1989), specifically ethane (C_2H_6) and carbon dioxide (CO_2). Nevertheless, the recovery rates obtained from Dunn et al. (1989) experiments did not correspond to those predicted by the model. Upon comparing their experimental rates with the model's predictions, they discovered that the coefficients of diffusion were significantly higher than the ones reported in the literature previously used in the analytical model. Even though these diffusion coefficients are far higher than the traditional diffusion coefficients found in the literature, Dunn et al. (1989) attempted to incorporate the convective dispersion concept into the conventional analytical framework to explain convective dispersion's impact on the solvent mass transfer process. This suggests that the first oil rate model developed by Butler and Mokrys (1989), which includes only the coefficient of molecular diffusion, cannot fully replicate the higher rate of recovery observed in porous rock. This indicates a need for further examination of potential additions to the mass transmission process, such as convective dispersion, to better reflect the high recovery rates obtained from experiments.

Das and Butler (1998) attempted to modify the earlier model of Butler and Mokrys (1989) by presenting the idea of porous media's apparent diffusion coefficient. This adjustment incorporated the solvent's inherent molecular diffusivity in bitumen (D_o), the porous media's porosity (Φ), and factor of cementation (Ω) giving Eq. (4) and resulted in the final form of oil flow rate in Eq. (5)

$$D_p = D_o \Phi^\Omega \quad (4)$$

$$q = 2L\sqrt{2kg\Phi^\Omega \Delta S_o N_s h} \quad (5)$$

However, Das and Butler (1998) carried out tests in sand packs utilizing heavy crude oils from Peace River and Athabasca, dissolved in butane as a solvent. Based on the experimental oil flow rates, they observed that the flow rate that was measured did not match the predictions made by their model introduced in Eq. (5). Upon the adjustment of the model to match the experiment results they found that the effective diffusivity (D_{eff}) was three to ten times higher than the documented one in the literature (D_s).

The recovery rate equation's dispersion term is crucial, as discussed earlier, and should be combined with the diffusion mechanism that is typically employed in these models. Additionally, further influencing factors must be fully comprehended and efforts should be made to discover alternative contributing processes in a particular VAPEX process instead of depending just on higher diffusivity coefficients to align with model and experimental data. Relying exclusively on these larger values might lead to unrealistic conditions or process definitions. Moreover, incorporating these mechanisms is a complex task.

2.1.3. Further VAPEX linear and curved developed analytical models

Moghadam et al. (2009) developed a linear mathematical model to forecast the total yield of oil. It was expected that the thickness of the transition zone, which is defined by two straight-line limits and lies between the solvent chamber and the untouched heavy oil/bitumen, remains constant. This thickness was used as a parameter to adjust to align the model with experimental outcomes. The variation in the zone of transition across different times showed a relative average error within 15% of the observed values from five VAPEX experiments. Further analysis led Moghadam et al. (2009) to recommend increasing the thickness of the transition zone as the permeability of the VAPEX model drops. Furthermore, the model indicated that falling and horizontal

spreading velocities would both diminish with time. Eq. (6) shows the total amount of oil produced during the spreading phase of the solvent chamber, and Eq. (7) gives the falling period of the solvent chamber boundaries cumulative oil production.

$$Q_{cs} = h^2 d \phi \Delta S_o \cot \theta \quad (6)$$

$$Q_{cf} = h^2 w d \phi \Delta S_o (2 - \cot \theta_s \tan \theta) \quad (7)$$

where d is the thickness of the transition zone.

Lin et al. (2014) formulated an analytical model that considers the circular shape of the solvent chamber. The model has three distinct stages of oil production which vary according to chamber stages including rising, spreading, and falling, represented by Eqs. (8)–(10), respectively, with the organized scenario depicted in Fig. 1. An adjustable parameter was the thickness of the transition zone between the heavy oil that was left undisturbed and the vapour solvent to align the model results with experimental VAPEX outcomes for each stage. During the stage of rising, its thickness was greater than that of the subsequent stages. It was also observed that this thickness increases as the permeability of the physical model decreases, consistent with the findings of Moghadam et al. (2009). They further elucidated that the single constant oil production rate projected by the first model by Butler and Mokrys (1989) was not a better representation of the VAPEX process than the several simulated oil rates employed in different stages. Consequently, Lin et al. (2014) suggested that the newly discovered model could serve as an additional tool for analyzing VAPEX process performance project visualization.

$$Q_r = \frac{\pi}{4} \Delta S_o \phi d H^2 \quad (8)$$

$$Q(t) = \Delta S_o \phi d H^2 \left(\frac{(\pi - \theta) + \sin \theta \cos \theta}{(1 + \cos \theta)^2} \right) \quad (9)$$

$$Q(t) = \Delta S_o \phi d w^2 \left[\frac{(\pi - \theta)}{4 \sin^2 \theta} + \frac{H}{w} - \frac{2 + \cos \theta}{4 \sin \theta} \right] \quad (10)$$

According to Lin et al. (2014) and Moghadam et al. (2009), Ma et al. (2017) disclosed that the earlier models corresponded to either linear or circular geometries for the solvent vapour chamber. However, the majority of VAPEX performances in the laboratory revealed that the forms were more like parabolic shapes. Thus, a parabolic shape-based analytical model, whether concave or convex was developed. In their experimental exercise, they digitized each solvent chamber image at different times to analyze the chamber shape by studying the sharp change in the grey level of each pixel. They compared the solvent chamber that is experiment digitized and the one predicted from the model by adjusting the zone of transition thickness. Compared to previous linear and circular models, their parabolic model more closely matches the

experimental VAPEX data. A constant value that does not change over time was used to define the transition zone, consistent with the definitions by Lin et al. (2014) and Moghadam et al. (2009). The variation in the thickness of the transition zone with time showed a relative average error of 13.1% when contrasted with the five VAPEX experimental thicknesses that were observed. However, Ma et al. (2017) assumed that the saturation of oil initially in the transition zone reduces to its saturation at a residual level as the zone of transition becomes part of the growing chamber of solvent, an assumption that requires further investigation.

Furthermore, Cheperli et al. (2022) developed a model defining the evolution of a solvent chamber during the stages of spreading and declining using an exponential function. To match the published VAPEX experimental data, they adjusted the transition zone thickness and the exponential function value. Relative average errors of 5.12% for the thickness of the transition zone and 7.73% for the predicted cumulative produced oil were found by analyzing four tests. This suggests that the model is more accurate than the linear and parabolic models, which had errors of 15% and 13.1%, as reported by Moghadam et al. (2009) and Ma et al. (2017) respectively, for the estimation of transition zone thickness. Additionally, the model predicts the oil production rate and matches the experimentally obtained solvent chamber shapes well with the corresponding cumulative oil production models expressed in Eqs. (11) and (12) for the concave and convex propagation respectively and the common production rate in Eq. (13) for both.

$$Q_o(t) = 2 \left(\frac{(H + a) \times (\ln(H + a) - \ln a) - H}{n} \right) \phi (S_{oi} - S_{or}) d \quad (11)$$

$$Q_o(t) = 2 \left(\frac{(a - H) \times (\ln(a - H) - \ln a) + H}{n} \right) \phi (S_{oi} - S_{or}) d \quad (12)$$

$$q_o(t) = \frac{d(Q_o(t))}{dt} \quad (13)$$

However, Cheperli et al. (2022) also developed models for the falling stage to estimate the cumulative diluted oil production rate for both concave and convex cases. They provided the corresponding cumulative oil production models in Eqs. (14), and (15) respectively, and the oil flow rate for both in Eq. (16).

$$Q_o(t) = 2 \left(\frac{HWn + aWn - ae^{wn} + a}{n} \right) \phi (S_{oi} - S_{or}) d \quad (14)$$

$$Q_o(t) = 2 \left(\frac{HW - aWn - ae^{wn} + a}{n} \right) \phi (S_{oi} - S_{or}) d \quad (15)$$

$$q_{ot} = \frac{d(Q_o(t))}{dt} \quad (16)$$

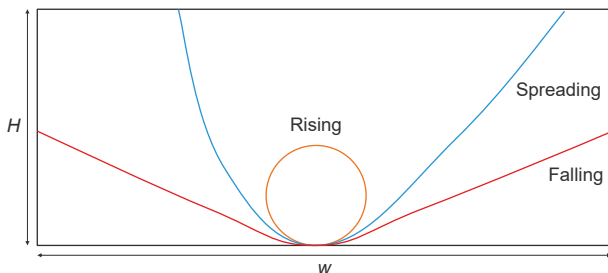


Fig. 1. Circular VAPEX solvent chamber formulation (Lin et al., 2014).

2.2. Warm VAPEX recovery mechanisms and analytical models evaluations

2.2.1. Warm VAPEX recovery mechanisms

The primary disadvantage of the VAPEX process is its slow oil production rate, which emanates from the inherently slower mass transfer compared to the heat of conduction and convection. However, warm VAPEX, which combines heating and solvent dilution, can significantly enhance the oil production rate (Rezaei and Chatzis, 2007). In warm VAPEX, the vapour solvent is heated

above the solvent dewpoint temperature under reservoir conditions where it condenses in situ after heat is lost to cold heavy oil or bitumen. The heated solvent transports some sensible heat to the VAPEX interface (the interface between liquid solvent and the initially heavy oil/bitumen in the reservoir), where it condenses on the surface of the bitumen/heavy oil and the liquid condensate effectively extracts/diluted the heavy oil or bitumen. The oil viscosity is reduced through solvent mixing with the bitumen/heavy oil via both diffusion and convection/dispersion, and latent heat transfer from the solvent to the bitumen under thermal diffusion.

2.2.2. Analytical flow rate models of the warm VAPEX injection

Motahhari and Khaledi (2018) developed a comprehensive analytical model for the gravity drainage process, incorporating key mass transfer mechanisms, including dispersion and diffusion, alongside heat transfer via conduction. Their model specifically accounts for the dependency of diffusion and dispersion coefficients on concentration, as well as on the properties of the fluid and rock, and drainage velocities. Employing a novel analytical approach, they solve the second-order nonlinear partial differential equation governing mass transfer within the mass boundary layer. Criteria are established to distinguish the mass transfer boundary layer from the overlying heat transfer boundary layer, collectively defining the drainage layer. The result is a closed-form analytical model that predicts the oil drainage rate due to gravity, heat, and dilution effects as a function of reservoir and fluid properties.

It is generally expected that two distinct boundary layers exist within the drainage layer because of the different mass and heat time scale transfer mechanisms. In the mass boundary layer, denoted as (E_C), the solvent concentration decreases from a maximum value (C_0) at the interface to a defined cut-off value (ξ_C). Within this layer, the mobilized oil exhibits reduced viscosity due to the combined effects of heat and dilution. Below the cut-off concentration, the solvent content does not significantly affect the viscosity of the solvent and bitumen mixture. In the heat boundary layer, denoted as (E_T), the temperature decreases from a maximum value (T_0) at the interface to the initial reservoir temperature (T_i). Theoretically, the heat boundary layer also encompasses the mass boundary layer. However, in their modelling study, the two boundary layers were separated and decoupled to bring the two effects fully analytical solutions for the mass and

heat transfer mechanisms. The two boundary layers are displayed in Fig. 2.

The complete model was derived based on the following assigned assumption for the systems (superheated vapour solvent, and bitumen):

- (1) The advancing bitumen-vapour front is characterized by a distinct gas/liquid interface, with the oil saturation behind this interface equating to the residual oil saturation.
- (2) The distribution of concentration and temperature ahead of the advancing front is steady-state profiles that remain constant over time.
- (3) The recovery process operates under isobaric (constant pressure) conditions. Therefore, the temperature and concentration at the interface are governed by the thermodynamic equilibrium between the gas and liquid phases for the operating pressure.
- (4) The solvent and bitumen are single-component hydrocarbons, so no heavy-phase precipitates or deposits form when they are combined to form a single phase (mixed).
- (5) There is no change in volume (shrinkage or expansion) of the oil when the solvent and bitumen are mixed.
- (6) Heat transfer is governed by conduction, while mass transfer is controlled by composition- and temperature-dependent diffusion, as well as velocity-dependent mechanical dispersion in the porous media.

The model was organized after analysis of the temperature profiles, concentration profiles from the interface to the dead heavy oil side, and mass transfer coefficients. Thereafter the analysis of the drainage rate flow accounted for two effects (heat and diffusion) was evaluated and finally, the total flow rate parameters insertion related to the one by Butler and Mokrys (1989) was established. The temperature distribution analysis follows equations of Fourier's second law which governs the one-dimensional heat flow by conduction in front of the moving solvent-bitumen front with the derivation procedures same as described by Zargar and Farouq Ali (2018). The transfer of mass ahead of the solvent moving front is controlled by Fick's law in Eq. (17), where further following the moving reference frame expressed in Eq. (18), the resulting is Eq. (19).

$$\frac{\partial}{\partial x} \left(K_D \frac{\partial C}{\partial x} \right) = \frac{\partial C}{\partial t} \quad (17)$$

$$\xi = x - \int_0^t U dt \quad (18)$$

$$\frac{\partial^2 C}{\partial \xi^2} + \frac{1}{K_D} \frac{\partial K_D}{\partial C} \left(\frac{\partial C}{\partial \xi} \right)^2 + \frac{U}{K_D} \frac{\partial C}{\partial \xi} = \frac{1}{K_D} \frac{\partial C}{\partial t} \quad (19)$$

Applying the boundary conditions given in Eqs. (20a) and (20b) and deriving the concentration-dependent coefficient of diffusion (K_D), combining an assumption of the steady state conditions, the solution of Eq. (19) is then arrived at in Eq. (21). However, considering the total mass transfer coefficient does not depend on the concentration of solvent ($K_D = k_0$), and the definition of ξ in Eq. (22), Eq. (21) is reduced to Eq. (23) which is analogue to the one of the temperature distribution by Butler (1985).

$$C = C_0, \quad \xi = 0 \quad (20a)$$

$$C = 0, \quad \xi = \infty \quad (20b)$$

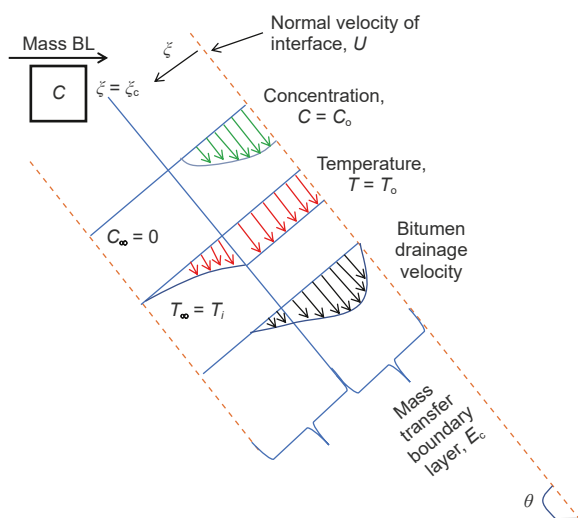


Fig. 2. The drained oil layer with magnified effects of mass transfer and heat transfer boundary layers (Motahhari and Khaledi, 2018).

$$K_D = \sum_{i=1}^n k_i \cdot C^i \quad (21)$$

$$\xi = -\left(\frac{k_o}{U} \ln\left(\frac{C}{C_o}\right) + \frac{1}{U} \sum_{i=1}^n \frac{k_i}{i} (C^i - C_o^i)\right) \quad (22)$$

$$C = C_o \cdot \exp\left(\frac{-U \cdot \xi}{k_o}\right) \quad (23)$$

The total mutual mass transfer coefficient (K_D), is the function of coefficients of mass transfer of diffusion and dispersion given in Eq. (24).

$$K_D = \frac{D_{SB}}{\tau} + \alpha_C \frac{V_{dl}}{\phi \cdot S_1} \quad (24)$$

where D_{SB} is the mutual diffusion coefficient of solvent and bitumen system; τ is the tortuosity of the porous media; α_C is the transverse dispersivity of the solvent and bitumen system; V_{dl} is the Darcy's velocity of the liquid hydrocarbon phase direction perpendicular to ξ ; S_1 is the saturation of the liquid hydrocarbon phase direction perpendicular to ξ .

According to [Butler and Mokrys \(1989\)](#), the drainage velocity of formed diluted oil (a composite mixture of solvent and bitumen) at any distance from ξ to the segment of the interface is then expressed in Eq. (25).

$$V_{dl} = k_{el} \cdot g \cdot \sin(\theta) \cdot \left(\frac{\rho_l - \rho_g}{\mu_l}\right)_{T,P,C} \quad (25)$$

where k_{el} is the effective permeability of the liquid hydrocarbon phase; ρ_l is the Density of the liquid hydrocarbon phase; ρ_g is the density of the gas phase (vapour solvent).

The flow rate in the drainage area is further expressed in Eq. (26). After replacing the definition of the drainage velocity in Eq. (26) using the mass transfer boundary and heat transfer boundary relationship respectively results in Eq. (27).

$$q_{oil} = \int_0^\infty V_{dl} \cdot (1 - C) d\xi \quad (26)$$

The factor $(1 - C)$ represents the exceptional of the volume fraction of the solvent

$$q_{oil} = k_{el} \cdot g \cdot \sin(\theta) \cdot \left[\int_0^{\xi_c} \left(\frac{\Delta\rho_{l-g}}{\mu_l}\right)_{C,T,o} \cdot (1 - C) \cdot d\xi + \int_{\xi_c}^\infty \left(\frac{\Delta\rho_{l-g}}{\mu_l}\right)_T \cdot d\xi \right] \quad (27)$$

where $k_{el} \cdot g \cdot \sin(\theta) \cdot \left[\int_0^{\xi_c} \left(\frac{\Delta\rho_{l-g}}{\mu_l}\right)_{C,T,o} \cdot (1 - C) \cdot d\xi \right]$ represents the flow rate contribution due to mass transfer; $k_{el} \cdot g \cdot \sin(\theta) \cdot \left[\int_{\xi_c}^\infty \left(\frac{\Delta\rho_{l-g}}{\mu_l}\right)_T \cdot d\xi \right]$ represents the portion of the flow rate that contributed by the heat of conduction.

Using the relationship of concentration and temperature with the distance from the interface as described, the transformation of the variable of integration from distance to concentration and temperature for the first and last integrals of Eq. (27) was done respectively yielding Eq. (28).

$$q_{oil} = \frac{k_{el} \cdot g \cdot \sin(\theta)}{U} (I_T + I_C) \quad (28)$$

where

$$I_C = \int_{C_{\xi_c}}^{C_o} \left(\frac{\Delta\rho_{l-g}}{\mu_l}\right)_{C,T,o} \cdot (1 - C) \cdot \left(\frac{k_o}{C} + k_1 + k_2 C + k_3 C^2\right) dC \quad (29a)$$

$$I_T = \int_{T_i}^{T_{\xi_c}} \left(\frac{\Delta\rho_{l-g}}{\mu_l}\right)_T \cdot \frac{\alpha_T}{T - T_i} dT \quad (29b)$$

Finally, the total flow rate can be estimated following the [Butler and Mokrys \(1989\)](#) thermal-solvent assisted gravity drainage system. The used assumption is that the drainage layers are attached continuously to the production well of length L . Therefore, the final form of production rate of oil becomes:

$$Q = 2L \sqrt{1.5 \phi \Delta S_1 (I_T + I_C) k_{el} g H} \quad (30)$$

A sophisticated estimation of the total production rate can be achieved using a semi-analytical sequential approach. This method involves dividing the reservoir height into small segments and tracking the location and inclination angle of the interface. Consequently, the I_T and I_C for each segment are calculated over small time steps.

Moreover, [Chai et al. \(2022\)](#) utilized the mixing rule of density and viscosity correlation Eqs. (31) and (32) respectively to provide the warm VAPEX velocity Eq. (34) after substitutions of those equations in velocity Eq. (33). The warm VAPEX flow rate model is given then in Eq. (35)

$$\rho_{mix,o}(\xi) = \rho_{o,ref} \left(1 - \frac{\alpha_T}{\rho_{ref}} T(\xi)\right) (1 - C_s(\xi)) + C_s(\xi) \rho_s \quad (31)$$

$$\mu_{mix,o}(\xi) = (\mu_{ref} (T_{inj} T_D(\xi))^{\beta_T})^{\alpha(1-C_D)} (\mu_{solvent}, T)^{(1-\alpha)C_D} \quad (32)$$

where $C_s(\xi)$ is the mass concentration of the solvent; C_D is the dimensionless concentration; ρ_s is the density of the liquid solvent; α is the factor to relate the composition factors considering density and viscosity of the component which is between 0 and 1; $\rho_{o,ref}$ and μ_{ref} is the reference density and viscosity of the heavy oil respectively; α_T and β_T is the density and viscosity coefficients variation with temperature respectively; $T_D(\xi)$ is the dimensionless temperature.

$$v_{max}(\xi) = \frac{k \Delta\rho(\xi) g \sin \theta}{\mu(\xi)} \quad (33)$$

$$V_{VAPEX}(\xi) = \frac{k \times k_e \left[\rho_{ref} \left(1 - \frac{\alpha_T}{\rho_{ref}} T\right) (1 - C_{s, mass}) + C_{s, mass} \rho_s \right] \times g \sin \theta \times \Delta S_{o, VAPEX} [1 - C_D]}{(\mu_{ref} (T_{max} \times T_D)^{\beta_T})^{\alpha(1-C_D)} (\mu_{SOLV}, T)^{(1-\alpha)C_D}} \quad (34)$$

$$q_{\text{VAPEX}} = \int_0^\infty V_{\text{VAPEX}}(\xi) \times L d\xi \quad (35)$$

The model validation using butane as the warm solvent with CMG STAR simulation was done as shown in Fig. 3 which shows a good match with the developed model underestimates the flow rate to some extent compared with the simulation results.

2.3. Hybrid warm VAPEX and steam injection recovery mechanisms and models evaluations

2.3.1. Recovery mechanisms of hybrid warm VAPEX and steam injection

The process is complex and involves simultaneous heat and mass transfer across the edges of the developed steam chamber. The heat transfer process is contributed much by steam and mass transfer is by the solvent (Kariznovi et al., 2014; Shaygan et al., 2022). The movement of the oil at the interface depends on the permeability and the oil's viscosity, which depends on temperature, solvent type, and solvent content. The solvent's mass transfer rate, however, is determined by solubility and diffusion processes the latter process measured by coefficients of dispersion. The solubility and diffusion are again affected by the system pressure, temperature and heavy oil viscosity (Keshavarz et al., 2015; Mworia et al., 2024; Sharma and Gates, 2011).

2.3.2. Analytical flow rate models of hybrid warm VAPEX and steam injection evaluation

Zargar and Farouq Ali (2021) developed a model that takes into consideration the impact of viscosity decrease by solvent dilution through diffusion phenomena and by heat through conduction phenomena. The injected solvent moves along with the steam in various directions, leading to an accumulation of solvent around the chamber's boundaries. The solvent accumulating in the edges condenses. The oil and condensed solvent combine at the advancing front via diffusion, yielding oil viscosity reduction. The accumulated solvent at the upper section of the reservoir is considered to remain in the vapour chamber (no diffusion).

The model was derived based on consideration of the temperature and concentration profiles from the interface perpendicular to the dead oil side with an assumption of a constant total rate of injection combining steam and solvent. The temperature distribution is obtained after the heat balance calculations while the solvent concentration distribution is obtained after the overall mass balance of the solvent. The calculations for the volumetric flow rate were

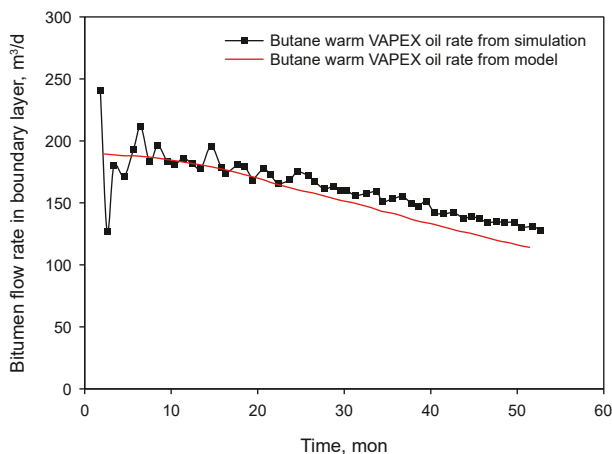


Fig. 3. Warm VAPEX flow rate validation with simulation utilizing butane as the warm solvent (Chai et al., 2022).

based on first the movement of the chamber of the steam interface by considering the growth of the steam chamber as an inverted triangle and secondly by the use of Darcy's law. Eq. (36) represents the movement of the vapour chamber interface. Eq. (34) gives the total latent heat of vaporization for both the solvent and the steam which is equal to the sum of the total heat gained to top overburdened rocks as lost heat (q_{loss}), heat that raises the saturated formation temperature to the steam temperature value (q_{sc}), and the heat gained into the zone of the oil ahead of the steam chamber (q_{res}).

$$U_f = \frac{1}{2} x_H' \sin(\theta_n)$$

where

$$\theta_n = \tan^{-1} \left(\frac{H}{x_n} \right) \quad (36)$$

$$q_{\text{inj}} = i_{\text{tot}} \left\{ \left(1 - C_{\text{sol}}^{\text{inj}} \right) f_s^{\text{st}} L_v^{\text{st}} + C_{\text{sol}}^{\text{inj}} f_s^{\text{sol}} L_v^{\text{sol}} \right\} \quad (37)$$

where i_{tot} is the total mass rate of steam and solvent; $C_{\text{sol}}^{\text{inj}}$ is the injected mass fraction of solvent; f_s^{st} and f_s^{sol} is the quality of injected steam and solvent respectively; L_v^{st} and L_v^{sol} is the latent heat of vaporization of the steam and solvent respectively.

$$q_{\text{res}} = q_{\text{inj}} - (q_{\text{loss}} + q_{\text{sc}}) \quad (38)$$

The injected solvent will have three distributions within the solvent-steam chamber which include the diffuse solvent in the oil, the retained solvent in the system, and the one accumulated at the top of the growth vapour chamber. The overall mass balance for the system of solvent in the reservoir is given in Eq. (39).

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{\partial m_{\text{sol}}}{\partial t} \quad (39)$$

$$\dot{m}_{\text{in}} = C_{\text{sol}}^{\text{inj}} i_{\text{tot}} \quad (40)$$

where $\dot{m}$ and m_{sol} are the solvent mass flow rate and accumulation in the system, respectively. As the injected solvent travels as a vapour phase along with steam in all directions, it condenses and accumulates at the vapour chamber boundaries once it reaches the necessary concentration and temperature for phase change. In the model, two key boundaries were identified including the vapour-oil interface and the top overburden rock. Both boundaries evolve, causing the vapour to lose heat at these points. Steam supplies heat to the system, while the solvent diffuses into the oil zone, diluting the bitumen. This combination of mechanisms lowers the bitumen's viscosity, enhancing recovery and overall energy efficiency. The solvent effectively reduces oil viscosity when it condenses at the vapour-oil interface. Therefore, condensation occurring elsewhere is considered a loss. The solvent mass balance is further expressed in Eq. (41).

$$\dot{m}_{\text{out}} = \dot{m}_{\text{top}} + \dot{m}_{\text{front}} \quad (41)$$

The relationship between the condensed solvent at the top and the front is given in Eq. (42)

$$\frac{\dot{m}_{\text{top}}}{\dot{m}_{\text{front}}} = \frac{x_H}{L_{\text{int}}} = \cos \theta_n \quad (42)$$

$$\dot{m}_{\text{out}} = (1 + \cos \theta_n) \dot{m}_{\text{front}} \quad (43)$$

According to the equation, when the process starts ($\theta_n = \frac{\pi}{2}$), no solvent is lost, and the whole part of the solvent undergoes

diffusion in heavy oil. However, as time progresses, the solvent loss process increases leading to a decrease in mass flux at the front.

$$\frac{1}{2}C_{\text{sol}}^{\text{inj}}i_{\text{tot}} - (1 + \cos \theta_n)\dot{m}|_{\text{front}} = \frac{1}{2}H\phi S_v \rho_{\text{sol}}^v x'_H, S_v = 1 - S_{\text{or}} \quad (44)$$

where $C_{\text{sol}}^{\text{inj}}$ is the mass fraction of the injected solvent and S_v is the depleted chamber's saturation. It is therefore presumed that the entire solvent condensed at the interface between vapour and oil can penetrate the oil zone, contributing to dilution of the bitumen. Fick's law can be used to compute the mass diffusion rate normal to the interface direction as given in Eq. (45).

$$\dot{m}|_{\text{front}} = -D_{\text{so}}A_{\text{int}}\rho_{\text{sol}}\left(\frac{\partial C}{\partial \xi}\right)_{\xi=0} \quad (45)$$

where C is the solvent volume fraction in the mixture.

For a unit-length horizontal well, the A_{int} can be expressed as:

$$A_{\text{int}} = 1 \times L_{\text{int}} = \frac{H}{\sin \theta_n} \quad (46)$$

Substitution of Eqs. (45) and (46) in Eq. (44) yields Eq. (47) for the solvent concentration gradient at the interface.

$$\left(\frac{\partial C}{\partial \xi}\right)_{\xi=0} = \frac{0.5}{(\csc \theta_n + \cot \theta_n)D_{\text{so}}\rho_{\text{sol}}H} \left\{ C_{\text{sol}}^{\text{inj}}i_{\text{tot}} - H\phi(1 - S_{\text{or}})\rho_{\text{sol}}^v x'_H \right\} \quad (47)$$

This concentration gradient Eq. (47) will serve as one of the Neumann boundary conditions for the concentration diffuse differential Eq. (49) with another boundary condition in Eq. (48) to give the solution of the pseudo steady state of the one-dimensional advection-diffusion equation with a constant coefficient of diffusion in the direction perpendicular to the interface of the vapour chamber, considering the moving boundary problem within diffusion velocity, U_f , which is expressed in Eq. (36) for the solvent concentration distribution ahead of the interface.

$$C(\xi \rightarrow \infty) = 0 \quad (48)$$

$$\frac{d^2 C}{d\xi^2} + \frac{U_f}{D_{\text{so}}} \frac{dC}{d\xi} = 0 \quad (49)$$

$$C(\xi, t) = \frac{1}{(1 + \cos \theta)\rho_{\text{sol}}x'_H} \left\{ \frac{C_{\text{sol}}^{\text{inj}}i_{\text{tot}}}{H} - \phi(1 - S_{\text{or}})\rho_{\text{sol}}^v x'_H \right\} e^{-\frac{1}{2D_{\text{so}}}\sin \theta x'_H \xi} \quad (50)$$

Displacement of volumetric oil resulting from the movement of the vapour chamber interface assuming the steam chamber grows in an inverted triangle shape, is calculated as follows.

$$q_o = \frac{1}{2}H\phi\Delta S_o x'_H \quad (51)$$

The small element of the oil flow rate parallel to the interface of vapour chamber can be expressed as:

$$dq_o = \frac{k_o \rho_o g \sin \theta_n}{\mu_o} d\xi \quad (52)$$

The oil density variation with temperature is assumed to be negligible compared to the viscosity change. The oil rate, as a function of temperature and concentration, and consequently the distance from the interface in the normal direction, for a particular interface position is given by

$$q_o = k_o \rho_o g \sin \theta_n \int_0^\infty \frac{1}{\mu_o(\xi)} d\xi \quad (53)$$

The equation for the viscosity of the mixture is then given as

$$\ln \mu_{\text{mix}} = \ln \mu_b - x_{\text{sol}} \ln \left(\frac{\mu_b}{\mu_{\text{sol}}} \right) \quad (54)$$

The main unknown in Eqs. (50), (51) and (53) is the maximum chamber velocity, x'_H . Since the equations are non-linear, the iterative method is then applied to calculate the maximum chamber velocity x'_H , which fits the equations.

No validation has been reported on the model but the comparison of the performances of the SAGD and SAGD with solvent assistance, such as the oil production rate, and cumulative steam-to-oil ratio (CSOR), by varying variable parameters and types of solvent in the model, was done. Fig. 4 displays the results of the oil production rate of various solvents (with the same mass fraction of 9%) with the higher oil production rate obtained by butane-steam co-injection.

Faradonbeh et al. (2012) developed a model that forecasted the improvement of the oil flow rate by the addition of solvent during steam-solvent SAGD. The model assumes that the steam chamber forms an inverted triangular shape with a lower vertex fixed at the production well under laterally spreading chamber stage consideration. The unsteady state distribution of solvent concentration upstream of the steam chamber is calculated using the Heat Integral Method (HIM), and the temperature profile is obtained under steady-state conditions. Utilizing these profiles of temperature and solvent concentration in the mobile zone, an appropriate correlation is used to compute oil viscosity, and the oil flow rate is determined by integrating across the whole mobile area. Because of the time scale differences between heat conduction and mass diffusion, there will be existing two-layer boundaries across the oil mobile zone. The border layer of solutes, represented by δ_L , is the thickness that, when exceeded the solvent concentration has no effect. In this layer, oil exhibits increased mobility as a result of the combined effects of heat and solvent on viscosity reduction. The boundary layer of heat, δ_T , represents the largest thickness layer at which temperature can be felt, which is around ten times thicker than the solute boundary layer.

The temperature distribution is the same as the one described by Reis (1992) as:

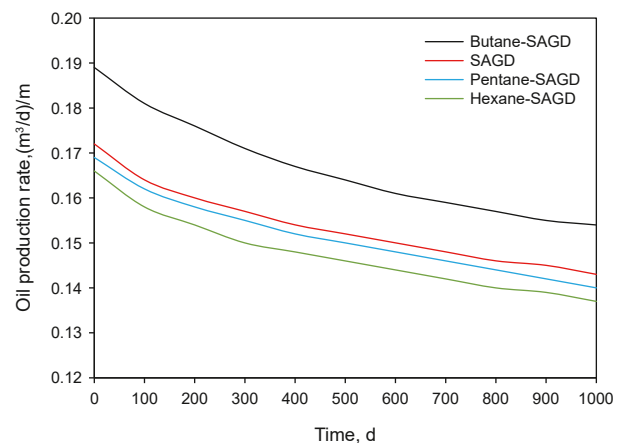


Fig. 4. Model application by production rate comparison between SAGD and variable warm VAPEX and steam co-injection (Zargar and Farouq Ali, 2021).

$$T_D = \exp\left(-\frac{aU_m}{\alpha}\xi\right) \quad (55)$$

The formed unsteady state concentration profile is given in Eq. (56):

$$\frac{\partial}{\partial \xi} \left[(D + \alpha_L U) \frac{\partial C}{\partial \xi} \right] + U \frac{\partial C}{\partial \xi} = \frac{\partial C}{\partial t} \quad (56)$$

Since Eq. (56) is non-linear, the concentration profile of the solvent is solved using HIM. In HIM, a polynomial or exponential solvent distribution function is chosen where the coefficients inside the function are estimated to satisfy conditions of boundaries. The depth of penetration of solvent is determined by solving an ordinary differential equation as a function of time derived from HIM. Eqs. (57a) and (57b) describe the concentration distribution in exponential and polynomial forms, respectively, ahead of the interface in dimensionless form as follows:

$$C_D = \exp\left(-\frac{\xi}{\delta_C}\right) \quad (57a)$$

$$C_D = \exp\left(1 - \frac{\xi}{\delta_C}\right)^n \quad (57b)$$

where δ_C is the depth of penetration of the solvent.

Variations in viscosity with temperature and concentration of solvent are associated with Eq. (58)

$$\frac{\nu_{\text{mix}}}{\nu_o} = \left[\frac{\nu_s}{\nu_o} \right]^{x_s} T_D^{-\bar{m}} \quad (58)$$

where ν_o and ν_s is the kinematic viscosity of the solvent and bitumen at steam temperature respectively; $\bar{m}$ is the equivalent exponent of viscosity of a bitumen-solvent mixture which is further defined in Eq. (59)

$$\bar{m} = x_s m_s + x_o m_o \quad (59)$$

where x_o and x_s are compositional parameters; m_o and m_s are the viscosity exponent of the solvent and pure bitumen, respectively with values between 3 and 4 for the m_o and close to unity for the m_s .

The oil flow rate for the small segment flow stream following the direction parallel to the interface can be calculated by Darcy's law with the exclusion of the solvent part as:

$$dq_o = \frac{k}{\mu_{\text{mix}}} \Delta \rho g \sin \theta (1 - C) d\xi \quad (60)$$

Integration of Eq. (60) after substitution of the viscosity mixture definition yields Eq. (61) for the total flow rate by heat and solvent dilution effect.

$$q_o = \frac{kg\alpha}{am_o\nu_o} \frac{\sin \theta}{U_m} q_R \quad (61)$$

And,

$$q_R = m_o \int_0^1 \left[\frac{\nu_o}{\nu_s} \right]^{x_s} [T_D]^{-\bar{m}-1} \left(1 - C_s T_D^{\frac{1}{aB_3Pe_r}} \right) dT_D \quad (62)$$

where q_R is the steam-solvent oil flow rate ratio at a certain point in time to the steam-only oil rate; C_s is the equilibrium

concentration of solvent at the contact/interface; γ is the dimensionless penetration depth; Pe is Peclet number respectively; B_3 is the dimensionless number depending on the bitumen's and the reservoir rock's physical characteristics as defined by Butler (1985).

No model validation process has been documented, but several process performance analyses using the model have been done with the following outcomes:

- (1) Expressing the concentration distribution in an exponential form yields predictions that closely match the analytical solution, compared to polynomial forms. The solvent penetration depth increases over time, and there is a sharp reduction in oil viscosity. Additionally, the production rate of oil is found to be proportional to the flow rate ratio square, based on integration across the mobile zone.
- (2) The enhancement of the oil flow rate in a hybrid steam-solvent process is compared to the steam-only recovery method based on various operational and fluid parameters. It is observed that the flow rate ratio remains nearly constant over time. Using heavier solvents improves the oil rate more than lighter ones, and the drainage rate increases linearly with the mole fraction of solvent in the vapour phase. The results indicate that the mechanical dispersivity factor is a significant parameter, as higher values lead to the oil rate reaching a maximum early in the solvent co-injection process.

Rabiei Faradonbeh et al. (2014) further developed a semi-analytical model, and extended it to include the addition of solvent during bitumen and heavy oil steam-solvent gravity drainage where the unsteady-state distribution of both temperature and solvent concentration ahead of the steam chamber is calculated using the HIM. Additionally, the model incorporates transverse dispersion and concentration-dependent molecular diffusion for the distribution of solvent. Oil viscosity is measured using profiles of the mobile zone's temperature and solvent concentration. The entire mobile zone is integrated to determine the oil flow rate. The analysis aims to develop a more comprehensive model that accounts for most phenomena that take place in the mobile zone. The recovery can be approximated using such a model and estimate the optimal operational parameters. The unsteady state dimensionless temperature and concentration distribution profiles are, therefore given in Eqs. (63) and (64) as follows:

$$\frac{\partial^2 T_D}{\partial \xi_D^2} + Pe B_3 \frac{\partial T_D}{\partial t_D} = B_3 \frac{\partial T_D}{\partial t_D} \quad (63)$$

$$\frac{\partial}{\partial \xi_D} \left[\left(\frac{e^{\eta^*} C_D}{Le B_3} + k_{t,D} Pe \right) \frac{\partial C_D}{\partial \xi_D} \right] + Pe \frac{\partial C_D}{\partial \xi_D} = \frac{\partial T_D}{\partial t_D} \quad (64)$$

where T_D is the dimensionless temperature; C_D is the dimensionless solvent concentration; $k_{t,D}$ and η^* are the diffusivity-concentration coefficient and dimensionless transverse dispersivity, respectively.

The HIM is again utilized to determine the temperature and concentration distribution profile. An exponential form of dimensionless temperature and solvent concentration distribution is employed as expressed in Eqs. (65a) and (65b)

$$T_D = \exp\left(-\frac{\xi_D}{\delta_{T,D}}\right) \quad (65a)$$

$$C_D = \exp\left(-\frac{\xi_D}{\delta_{C,D}}\right) \quad (65b)$$

The corresponding dimensionless penetration depth for the temperature and solvent are given in Eqs. (66a) and (66b):

$$\frac{\partial \delta_{T,D}}{\partial t_D} = \frac{2}{\pi} \left(\frac{1}{B_3 \delta_{T,D}} - P_e \right) \quad (66a)$$

$$\frac{\partial \delta_{C,D}}{\partial t_D} = \frac{1}{\delta_{C,D}} \left(\frac{F(\eta^*)}{L_e B_3} - P_e \right) - P_e \quad (66b)$$

The viscosity of the fluid mixture in the system is the same as that derived by Faradonbeh et al. (2012). Therefore substitution of the expression for the viscosity mixture and integration over the entire integral of the oil flow rate as given in Eq. (60), the final form of the dimensionless total flow rate is given in Eq. (67)

$$q_{o,D} = \frac{q_o}{\alpha_m \phi \Delta S_o B_3} = \delta_{T,D} B_3 \sin \theta q_R \quad (67)$$

where $\delta_{T,D}$ is the dimensionless thermal penetration depth and

$$q_R = m_o \int_0^1 \left[\frac{\nu_o}{\nu_s} \right]^{x_s} [T_D]^{\bar{m}-1} \left[1 - C_s T_D^{\frac{\delta_{T,D}}{\delta_{C,D}}} \right] dT_D \quad (68)$$

Fig. 5 shows a comparison of oil production rates as predicted by the CMG STAR simulator and the model which is the model validation process. In this scenario, hexane, constituting 5% by volume, is injected with steam at a pressure of 2.5 MPa and a temperature of 224 °C. An initial increase in the oil production rate is observed, peaking at approximately 115 m³/d. After about a year, the oil production rate declines to around 100 m³/d, followed by a sharp decrease over a short period. The model generally underestimated the simulation oil production rate outputs.

3. Non-condensable gas and steam injection

3.1. Recovery mechanisms of hybrid steam and non-condensable gas injection

Steam is simultaneously injected into the reservoir along with a tiny amount of NCG. This process stimulates the steam chamber's

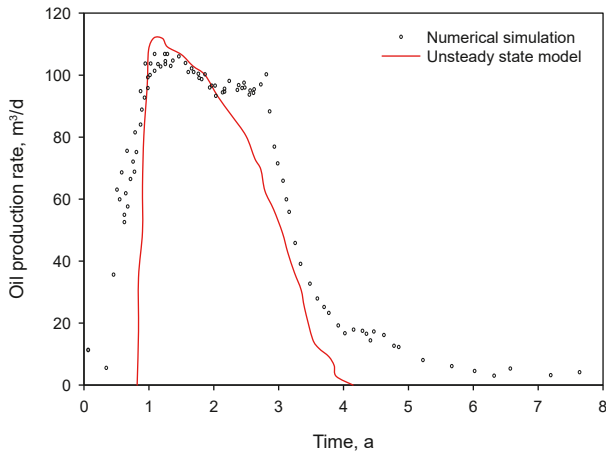


Fig. 5. Comparison of the flow rate of the unsteady state analytical solvent-steam co-injection model by Rabiei Faradonbeh et al. (2014) and simulation (Rabiei Faradonbeh et al., 2014).

lateral expansion, which improves the final oil recovery. The NCG has low heat conductivity, low viscosity, and high oil solubility. It builds up at the steam chamber's top due to its lower density compared with steam condensate. This accumulated gas reduces heat loss and steam consumption, preserving the steam chamber's pressure, and subsequently raising the rate of oil extraction/recovery. In particular, the nitrogen co-injected acts as an insulator of steam heat, improving overall thermal efficiency, while nitrogen fingering enhances the steam flow capacity. The co-injected CO₂ will also reduce the oil viscosity after dissolving into it and lower the oil-water interfacial tension by breaking the water-in-oil emulsion, thereby improving oil mobility because of its high solubility capacity compared with nitrogen in heavy oil (Wang et al., 2023, 2025). Both CO₂ and nitrogen processes are present in the co-injected flue gas, while CO₂ has a less potent heat-insulating effect than nitrogen.

3.2. Analytical flow rate modelling of hybrid steam and non-condensable gas injection

Sharma and Gates (2011) used the one-dimensional Fourier equation to define the conduction heat flow of the moving chamber for the linear geometry non-condensable-steam vapour interface as:

$$\alpha \frac{\partial^2 T}{\partial \xi^2} - U \frac{\partial T}{\partial \xi} = \frac{\partial T}{\partial t} \quad (69)$$

The defined temperature profile which is normal from the interface to the dead heavy oil/bitumen side is expressed based on Reis (1992) explanation. Because NCG and steam vapour coexist within the chamber's gas phase, the steam partial pressure determines the temperature of the chamber depending on the fraction of moles of NCG injected following Dalton's Law as:

$$p_s = (1 - y_g) p_i \quad (70)$$

where p_s is the steam partial pressure; p_i is the system total pressure; y_g is the mole fraction of the non-condensable gas.

The rate of change of the NCG mole fraction (y_g) under a pseudo-steady state is presented as

$$\frac{dy_g(\xi, t)}{dt} = \frac{\partial y_g}{\partial \xi} U \quad (71)$$

The condensation rate of steam can be determined by assessing the change of rate in partial pressure of steam within the zone of oil which is mobile represented by the correlation of temperature-pressure of saturated steam given in Eq. (72).

$$\log_{10} p_s = 9.8809 - \left[2.42223 + \frac{326.57}{T + 273.15} \right]^2 \quad (72)$$

Because gravity, not pressure gradients, is the primary force behind fluid production, operational pressures are frequently maintained around reservoir pressures. As a result, pressure gradients are generally negligible, less than 5 kPa/m, at the chamber's periphery. Similar results were seen while using numerical simulations. In the fully developed steam chamber, pressure can be considered to be constant within the mobile zone and the correlation between steam partial pressure and gas mole fraction is expressed in Eq. (73).

$$\frac{dy_g}{d\xi} = -\frac{\partial}{\partial \xi} \left(\frac{p_s}{p_i} \right) \quad (73)$$

The steam condensation-induced concentration gradient of NCG results in its counter-current diffusion toward the chamber.

Fick's first law, which, in quantitative terms, describes this process in limitation to convection, is expressed as follows:

$$\frac{\partial C_{\text{gas}}}{\partial t} = D_{\text{gas,eff}} \frac{\partial^2 C_{\text{gas}}}{\partial x^2} \quad (74)$$

where $D_{\text{gas, eff}}$ is the coefficient of effective diffusivity that takes into account the molecular diffusion and convective dispersion; C_{gas} is the NCG's molar concentration in the gas phase.

Diffusion equations can be determined similarly to temperature equations by transferring the frame of reference to the moving chamber interface and assuming pseudo-steady-state expansion of the system. The NCG accumulation close to the edges of the chamber seizes and the saturation of the gas drops to zero after the counter-current rate of diffusion surpasses the rate of steam condensation. The measurement of the distance from the chamber edge at which the gas phase saturation approaches zero and the NCG concentration in the gas phase peaks is referred to as ξ^* . On the assumption that instantaneous phase equilibrium is achieved, the NCG is divided into the oil and gas phases. The equilibrium K -values calculate the mole fraction of NCG that enters the oil phase and dissolves at the condensing interface.

$$K = \frac{y_g}{x_g} \quad (75)$$

where y_g is the mole fraction in the gas phase of the non-condensable gas; x_g is the mole fraction of the non-condensable gas in the diluted oil (oleic) phase.

The K values are usually empirical values correlated to pressure and temperature.

$$K = \left[\frac{k_{v1}}{p_i} + k_{v2}p_i + k_{v3} \right] e^{(k_{v4})/(T-k_{v5})} \quad (76)$$

where k_{v1} through k_{v5} are the constant coefficients.

Since the overall mobile zone pressure stays relatively constant, the profile of temperature obtained has the greatest influence on the solubility. NCG solubility in oil and its mole fraction in the gas phase rise in the mobile zone as temperatures drop, up to a distance ξ^* , after that, the gas phase saturation falls to zero. Dispersive and diffusive transport allows the non-condensable gas that dissolved in oil to continue to mix with the oil phase beyond, ξ^* , i.e. $\xi > \xi^*$. Similar to how in the gas phase, the diffusion is quantitatively characterized in Eq. (77), this process can be expressed as follows:

$$U \frac{\partial C_{\text{oil}}}{\partial \xi} = -D_{\text{oil,eff}} \frac{\partial^2 C_{\text{oil}}}{\partial \xi^2} \quad \text{for } \xi > \xi^* \quad (77)$$

where C_{oil} is the injected NCG's molar concentration in the oil phase; $D_{\text{oil, eff}}$ is the effective diffusion coefficient in the oil phase, which takes hydrodynamic dispersion and molecular diffusion into account.

Temperature, viscosity, molecule weight, and gas concentration all affect the molecular diffusion coefficient. Nonetheless, it was believed that the fluctuation of $D_{\text{oil, eff}}$, with the concentration of NCG would be minimal in the proposed model. Oil saturation in the mobile zone varies from the initial saturation found further within the cold reservoir to residual levels near the chamber's edge. The oil saturation is assumed to vary linearly with normalized temperature as expressed in Eq. (78).

$$S_o = (1 - T^*)(1 - S_g - S_{\text{wr}}) + S_{\text{or}} \quad (78)$$

In the case of this modelling T^* , is estimated as:

$$T^* = \frac{T - T_s/2}{T_s - T_s/2} \quad (79)$$

This approximation in Eq. (79) is corroborated by the fine-grid numerical simulation observations study conducted by Sharma and Gates (2011), which demonstrates as the temperature drops, the oil's saturation rises linearly. The saturation of gas within the vapour chamber will be given as:

$$S_{\text{gi}} = 1 - S_{\text{or}} - S_{\text{wr}} \quad (80)$$

When the temperature drops and steam condensation starts beyond the chamber edge, gas saturation progressively drops from S_{gi} , until it reaches zero at ξ^* . Therefore, from $\xi^* = 0$, to $\xi = \xi^*$, the assumption is that gas saturation will vary linearly from $S_g = S_{\text{gi}}$ to $S_g = 0$ and can be expressed as:

$$S_g = S_{\text{gi}} \left(1 - \frac{\xi}{\xi^*} \right) \quad (81)$$

Beyond the gas chamber's edge, the water saturation can be expressed as

$$S_w = 1 - S_g - S_o \quad (82)$$

Phase mobility, or the ratio of effective permeability to viscosity, or λ , for each phase, determines how the phases of oil, water, and gas move within the reservoir which is a function of the relative permeability, k_r highly dependent on the saturation of the phase. The mobile zone's multiphase movement of oil, water condensate, and gas requires the application of three-phase relative permeability systems as expressed in Eqs. (83), (84), and (85) respectively.

$$k_{\text{ro}} = k_{\text{roCW}} \frac{S_o^3 (1 - S_g + S_w - 2S_{\text{wr}})}{(1 - S_{\text{wr}})^4} \quad (83)$$

$$k_{\text{rw}} = k_{\text{rwro}} \frac{S_o^3 (S_w - S_{\text{wr}})^4}{(1 - S_{\text{wr}})^4} \quad (84)$$

$$k_{\text{rg}} = k_{\text{rgcw}} \frac{S_g^3 (2 - S_g - 2S_{\text{wr}})^4}{(1 - S_{\text{wr}})^4} \quad (85)$$

The oil flow rate in the presence of NCG can then be evaluated based on the mobility profiles. According to Darcy's law for gravity flow, the oil drainage rate for a section of thickness $d\xi$ and length L is as follows:

$$dq = \frac{(d\xi \times L) k_o \Delta \rho g \sin \theta}{\mu_o} \quad (86)$$

Because of the big difference between the density of oil and gas, then $\Delta \rho$ can be replaced by the ρ_o in Eq. (86) as

$$dq = \frac{(d\xi \times L) k_o \rho_o g \sin \theta}{\mu_o} \quad (87)$$

Eq. (87) can then be rewritten when including the kinematic viscosity as:

$$dq = \frac{L k_o g \sin \theta}{\nu_s} e^{-(mU\xi)/(\alpha)} d\xi \quad (88)$$

The oil, water, and gas phase saturations which are determined by how far away the SAGD chamber's edge is, affect the effective permeability of the oil, k_o . Eq. (88) can therefore first be integrated by replacing the effective oil permeability definition with oil relative permeability, k_r , and the total average reservoir

permeability, k , from the interface to the far point where the temperature of the oil is the initial reservoir temperature as:

$$q = \frac{Lkg \sin \theta}{\nu_s} \int_{\xi=\infty}^{\xi=0} k_{ro} e^{-(mU\xi)/(\alpha)} d\xi \quad (89)$$

The mass balance around the differential slice can be obtained as long as there is little variation in the oil phase density considering the geometry point of view can be expressed as:

$$\frac{U}{\sin \theta} = \frac{\cot \theta}{\Phi \Delta S_o L} \frac{\partial q}{\partial x} = \frac{\partial q}{\partial z} \quad (90)$$

Substitution of Eq. (90) in the flow rate Eq. (89) yielding the final expression for the oil flow rate is given in Eq. (91).

$$q = 2L \sqrt{\frac{2\alpha hkg\Phi \Delta S_o}{m\nu_s} \left[\int_{\xi=\infty}^{\xi=0} k_{ro} e^{-\bar{\xi}} d\bar{\xi} \right]} \quad (91)$$

where

$$\bar{\xi} = \left(\frac{mU\xi}{\alpha} \right) \quad (92)$$

4. Challenges, research gaps and perspectives

4.1. Challenges

More than one vapour chamber shape has been developed which includes linear, circular, parabolic, and exponential. Sensitivity analysis studies of the models that can fit a specific heavy oil reservoir are the crucial task due to different growing vapour chamber features that define the shape of the interfaces between vapour chambers and dead heavy oil. Therefore, this understanding must occur before driving model identification for a given heavy oil reservoir. The goal is to obtain precise and reliable oil flow rate estimation results. This requires different mathematical and reservoir physical process interrelationship techniques which retains as a challenging modelling task. Apart from that, the chamber growing stage in the models is rising, spreading, and declining as documented by Butler (1985); (Mworio et al., 2024; Zargar and Farouq Ali, 2018; Zargar et al., 2020) need also to be well modelled in the equations to have a complete model that mimics the oil drainage process which will continue to remain as the challenge in the process of timing these stages in the comprehensive model's formulation.

Some of the reviewed models have not been documented to be validated including the ones by Faradonbeh et al. (2012), Motahhari and Khaledi (2018) and Zargar and Farouq Ali (2021). They have been derived and directly used for prediction, which is one of the recognized challenges since model verification and validation are recommended tasks after model formulation to have a complete set of model analysis. This helps escape the challenges of significant model output inconsistencies in process prediction. Because it will be easy to see the model prediction result differences in comparison with other reliable performance tools, including experiment, field, and simulation. Hence, estimation of error factors and model parameter tuning efforts can be made to fall on the values of the reliable performance tools results.

The concept of constant steam injection temperature in all parts of the steam chamber periphery remains uncertain as

demonstrated by the temperature variations along the steam chamber edges in the model of Shi and Okuno (2018), which becomes more closely to the simulation model results of Sheng (2013) compared with all derived reviewed models which of course assuming the temperature of the steam chamber edges is constant and equals to steam temperature. This challenges the subsequent modelling to consider the steam chamber edge temperature variation to mimic drainage phenomena to have a more accurate predicting model.

4.2. Research gaps and perspectives

Convective heat transfer in conjunction with the heat of conduction in the models derivation is one of the gaps, since the majority of models utilized only conduction heat transfer phenomena which emphasizes more research combining these two heat transfer mechanisms in the further.

For simplicity, most of the equations are derived based on the steady-state flow conditions for the VAPEX, warm VAPEX, hybrids of warm VAPEX and non-condensable gases with steam. Moreover, understanding of the boundaries of the mass transfer and heat transfer in the warm VAPEX, and the hybrid of the warm VAPEX and steam remain uncertain, hence this gap enhances more modelling studies to understand the heat and mass transfer boundaries (velocities) differences vs. heavy oil viscosity reduction and hence clearly analytical oil flow drainage rate estimation. Future research models concentrated on curved shapes are required to have expanded modelling for these processes during the field history match for model selection. Moreover, differential chamber growing stages include rising and decline stages remaining as the gap.

Validation exercises of the models by experiments, simulation and field studies were conducted. The linear warm VAPEX model by Chai et al. (2022) and the linear model of the hybrid of warm VAPEX and steam injection by Rabiei Faradonbeh et al. (2014) generally underestimate the simulation results. The process of uncertainty parameters estimation in the models remains as the gap since few model uncertainty parameters sensitivity analyses have been conducted. As a result, the remaining derivatives have not been matched with experiments, simulations, and field results except for VAPEX derivative models of Moghadam et al. (2009), Lin et al. (2014) and Cheperli et al. (2022) which sensitively turning the thickness of the drainage boundary layer between the vapour chamber and dead heavy oil of the models to match with the experimental oil flow rate outcomes.

5. Conclusion and recommendations

A critical review of oil flow rate modelling and its substitutes is presented in this paper. It overviews the engineering physical principles used in the derivation, geometries used for defining the shape of the steam/solvent/non-condensable gases chamber, and chamber growth stages. Moreover, model predictions and validation exercises were analyzed. Important issues that were recognized include the following:

- (1) VAPEX, warm VAPEX, and hybrids of warm VAPEX and NCG have additional mass balance theory including diffusion and dispersive mass transfer coefficients, because of solvent diffusion in heavy oil/bitumen. All systems were based on steady-state conditions with a portion of unsteady state noted in the derivative of hybrid warm VAPEX and steam, which stimulates further research to include unsteady state analysis in other derivatives to capture different flow

scenarios that reflect actual processes targeting to the more accurate and reliable models.

- (2) Warm VAPEX and NCG were noted to be the matured systems due to a small number of modelling with only two linear model studies found for warm VAPEX and one for hybrid NCG with steam, which is an attractive future area of modelling. Hybrids of warm VAPEX and NCG with steam injection models were only characterized by linear geometries which impulse further studies on curved systems.
- (3) Linear models overestimated the flow rate compared with experimental results, and the curved models underestimated the flow rate compared with the field results. A lack of validations was recognized in the hybrids developed models of warm VAPEX and NCG with steam. No uncertainty parameters values switching sensitivity processes were carried out to ensure that the models matched fields, experimental, and simulation results except for some models under the VAPEX process. Future research focuses on parameter-switching sensitivity analysis to identify the most significant uncertainty factors in the equations and exploration of other driving mechanisms are therefore recommended.
- (4) Re-consideration of the sensible heat of the water condensate and condensed solvent is recommended in future studies in the constant rate of steam injection heat balance equation, since only latent heat was considered while steam and solvent condensing during the process.

CRediT authorship contribution statement

Zheng-Bin Wu: Supervision, Conceptualization. **Morice Richard Mworio:** Writing – review & editing, Writing – original draft, Software, Methodology, Conceptualization. **Xiao-Hu Dong:** Writing – review & editing. **Kun Shu:** Resources, Investigation. **Shu Jiang:** Supervision, Conceptualization.

Conflict of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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