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A physics-informed adaptive radial basis neural network (PIARNN) for fractured horizontal well simulation

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ABSTRACT

Multistage fractured horizontal wells are widely applied to develop low-permeability reservoirs, where precise numerical simulation plays a vital role in reservoir management. However, the governing equations of such fractured systems exhibit strong nonlinearity and are typically solved using the Newton–Raphson method. This approach requires repeated assembly and inversion of large Jacobian matrices, resulting in prohibitive computational cost. Recently, physics-informed neural networks (PINNs) have emerged as a promising approach for solving nonlinear partial differential equations. Nevertheless, most existing PINN-based methods focus on vertical well scenarios and directly learn the full pressure field, which limits computational efficiency. To address these limitations, this paper proposes a physics-informed adaptive radial basis neural network (PIARNN) for fractured horizontal wells simulation. The method achieves Newton-level accuracy without explicit Jacobian computation. Specifically, the Embedded Discrete Fracture Model (EDFM) is discretized and incorporated into the loss function to impose physical constraints. An adaptive factor is introduced, and a dual-branch architecture is designed to independently characterize matrix and fracture pressure dynamics. Furthermore, radial basis functions are used for feature mapping, and model learns pressure increments at the next time step to approximate the Newton iteration updates. Three reservoir cases are used for model validation. The results demonstrate that PIARNN can accurately predict pressure fields and bottomhole pressure responses with errors stabilized at the order of 10^{-5} . The model exhibits adaptability to both hydraulic fractures and discrete natural fractures. Compared with existing PINN and PICNN methods, PIARNN achieves higher accuracy and superior stability in complex fracture systems. This study provides a promising alternative to traditional Newton-based solvers for nonlinear seepage problems in low-permeability reservoirs.

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1. Introduction

In recent years, artificial intelligence has attracted increasing attention due to its powerful computational capabilities (Pilz et al., 2025). One prominent application is surrogate modeling, which has emerged as an efficient choice for engineering problems by directly learning patterns from data to replace numerical computation (Hui et al., 2025; Chen et al., 2025). While sacrificing some accuracy compared to traditional numerical methods, it

allows for rapid modeling across many industries, including but not limited to aerospace (Sunny, 2025), materials science (Bai and Zhang, 2025), fluid dynamics (Zhang et al., 2025), and reservoir simulation (Kandala et al., 2025). Existing methods can generally be classified into three categories: data-driven methods, physics-driven trained with labels methods, and purely physics-driven approaches. Data-driven methods enable rapid model construction and accurate prediction based on large datasets; however, they rely heavily on high-quality samples, which limits their applicability in practical engineering scenarios. By incorporating physical constraints, the dependence on data can be significantly reduced, allowing neural networks to learn underlying physical laws from limited samples rather than merely fitting observed data. This facilitates high-accuracy modeling with scarce data and

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provides a promising route for efficient forward modeling in engineering applications (Cui et al., 2026).

Physics-driven approaches have become a recent research focus, as represented by Physics-Informed Neural Networks (PINNs) (Raissi et al., 2019). In the oil and gas domain, early physics-constrained methods relied on automatic differentiation (AD) to approximate partial derivatives and construct PDE-constrained loss functions for neural network training. This approach has been applied to solve the Buckley–Leverett equation for non-piston waterflooding and diffusion equations governing fluid transport (Fuks and Tchelepi, 2020; Zhang et al., 2024). By generating abundant high-quality sample points or developing sophisticated constraint algorithms, notable progress has been made in addressing gradient-shock-type problems (Badawi and Gildin, 2023; Wang et al., 2025). However, such approaches remain computationally cumbersome, challenging to implement, and difficult to scale to high-precision physics-constrained models. In addition, due to abrupt gradient variations near boundaries, AD often struggles to accurately capture sharp gradient features (Han et al., 2023). This limitation has motivated the adoption of finite volume methods (FVM) to replace traditional point-wise constraint strategies, thereby facilitating more robust modeling. This shift has further advanced the development of intelligent reservoir simulation frameworks.

In this context, Zhang et al. (2023) first proposed combining Physics-Informed Convolutional Neural Networks (PICNN) with finite volume methods to solve oil-water two-phase flow problems. Subsequently, Chen et al. (2023) extended this framework within a fully implicit formulation, enabling the simultaneous update of pressure and saturation. Li et al. (2024) further integrated this approach with local grid refinement techniques and proposed the LCNN model. More recently, Lv et al. introduced the Physics-Informed Residual Network (PIResNet) (Lv et al., 2024) and the Physics-Informed Radial Basis Function Neural Network (PIRBFNN) (Lv et al., 2025). By incorporating residual connections and radial basis functions, these models facilitate accurate gradient propagation and significantly improve the computational efficiency of CNN-based approaches. Existing studies have achieved initial progress in modeling conventional waterflooded reservoirs, providing valuable insights for intelligent reservoir simulation. Nevertheless, these approaches cannot be readily extended to low-permeability reservoirs stimulated by multistage fractured horizontal wells (MFHW). Hydraulic fracturing generates coupled artificial-natural fracture networks with complex flow channels, raising barriers to high-precision reservoir modeling. Notably, MFHW dominate low-permeability reservoirs exploitation, simulating complex fracture networks restricts the application of intelligent modeling.

From the perspective of fracture simulation, the early Warren–Root model provides a simple and feasible approach (Warren and Root, 1963). Based on the continuum medium assumption, this model uses matrix blocks and fissures to characterize the reservoir space and fractures seepage pathways. Nevertheless, it is overly idealized and fails to describe multi-scale fractures and highly conductive fractures formed by artificial fracturing. Subsequently, Noorishad and Mehra (1982) proposed a Discrete Fracture Model (DFM) for accurately characterizing fracture using finely subdivided meshes. Unfortunately, DFM is computationally expensive, restricting its application in large-scale field problems. In contrast, the Embedded Discrete Fracture Model (EDFM) embeds fractures into the background matrix grid without requiring local grid refinement, thereby striking a favorable balance between computational efficiency and modeling fidelity (Li and Lee, 2008).

As such, EDFM has become a preferred approach for the simulation of fractured reservoirs under complex geological conditions. Despite its attractive merits, the integration of EDFM with data-driven or physics-constrained intelligent models remains non-trivial. Several recent studies have attempted to bridge this gap. For instance, Yan et al. (2024) developed an E-PINN framework by incorporating grid information matrices into neural networks, enabling physics-constrained modeling of fractured systems based on EDFM. Lin et al. (2025) further extended this framework to enable multi-factor dynamic learning, thus enhancing model adaptability. Nevertheless, these methods typically adopt a time-marching scheme, where the prediction at each time step relies on outputs from prior steps, which may induce cumulative error propagation. Moreover, they mainly consider simplified fracture patterns and rarely involve realistic MFHW. Consequently, the development of an EDFM-based physics-constrained modeling approach that can accurately capture complex fracture networks while retaining high computational efficiency is highly desirable for MFHW reservoir simulation.

In this study, we propose a physics-informed adaptive radial basis neural network (PIARNN) for EDFM-based modeling. The objective is to enable large-scale, physics-constrained surrogate simulations. Unlike conventional PINN methods, the proposed approach does not require grid values from the current time step as inputs. Instead, it employs an iterative neural network framework to obtain accurate solutions. This design alleviates error accumulation during time-marching.

First, the mathematical formulation of EDFM is derived to explicitly describe fracture geometries. Then, the governing PDEs for fracture-matrix flow interactions are constructed and incorporated as physical constraints for the model's training. By combining EDFM with the proposed network, the method can efficiently and accurately simulate production from hydraulically fractured horizontal wells.

The remainder of this paper is organized as follows. Section 2 presents the discretized PDE formulation of EDFM and Section 3 elaborates on the structure and principles of PIARNN. Section 4 evaluates the accuracy and generalization ability of the method using three representative hydraulic fracturing cases. Section 5 discusses the advantages of each module in PIARNN. Section 6 concludes the work and outlines future research directions.

2. Governing equations

2.1. Governing flow equations

Based on the continuity equation, the mass conservation for the matrix and fracture system of single-phase fluid is Eq. (1).

$$\begin{cases} \frac{\partial}{\partial t}(\phi_m \rho) + \nabla \cdot (\rho \mathbf{v}_m) = \rho (q_{ms} - q_{mf}) / V_m \\ \frac{\partial}{\partial t}(\phi_f \rho) + \nabla \cdot (\rho \mathbf{v}_f) = \rho (q_{fs} - q_{fm} - q_{ff}) / V_f \end{cases} \quad (1)$$

Here, ρ is the fluid density, kg/m^3 ; q_{ms} is the source-sink term of matrix system, $\text{m}^3 \cdot \text{s}^{-1}$; ∇ is the gradient operator; ϕ_m is the porosity; t is time, s ; $\mathbf{v}_m$ is the flow velocity of fluid in the matrix system, $\text{m} \cdot \text{s}^{-1}$; q_{mf} is the cross-flow rate from the matrix system to the fracture system, $\text{m}^3 \cdot \text{s}^{-1}$; V_m is the volume of matrix grid, m^3 ; ϕ_f represents the fracture porosity; $\mathbf{v}_f$ represents the fluid flow velocity in the fracture system, $\text{m} \cdot \text{s}^{-1}$; q_{fs} is the source-sink term of fracture system, $\text{m}^3 \cdot \text{s}^{-1}$; q_{ff} represents flow between intersecting fractures, $\text{m}^3 \cdot \text{s}^{-1}$; V_f represents the volume of fracture grid, m^3 .

The volume factor is calculated as follows:

$$B = \frac{\rho_{sc}}{\rho} \quad (2)$$

where B is the fluid formation volume factor, m^3/m^3 ; ρ_{sc} is the fluid density under standard surface conditions, kg/m^3 .

Based on Darcy's law, the flow velocity equation is expressed as:

$$\nu_\alpha = -\frac{K_\alpha}{\mu} (\nabla p_\alpha - \rho g \nabla D) \quad (3)$$

where with $\alpha \in \{m, f\}$, m = matrix grid, f = fracture grid; K_α is the permeability, m^2 ; μ is the fluid viscosity, $\text{Pa}\cdot\text{s}$; p_α is the pressure, Pa ; g is the gravitational acceleration, m/s^2 ;

EDFM is adopted to explicitly characterize complex fracture geometries. Within EDFM, the flux exchange is realized through indirect conduction, which are called non-neighboring connections (NNCs). As illustrated in Fig. 1, the three types of NNCs include: (1) NNC between fracture and matrix systems (NNC1); (2) NNC between different intersecting fractures (NNC2); (3) NNC between segments of the same fracture (NNC3).

The NNC flow rate is given by Eq. (4).

$$q_{\text{NNC}} = \sum_{nnc \in \{f, fm, ff\}} \frac{T_{\text{nnc}}}{\mu} (p_j - p_i) \quad (4)$$

where T_{nnc} represents NNCs transmissibility, m^3 ; p_j and p_i represent the pressures of NNC grids and the local grid, respectively, Pa .

For the first NNC, the transmissibility between matrix and fracture is defined as:

$$T_{\text{mf}} = \frac{2A_f k_{\text{mf}}}{d_{\text{mf}}} \quad (5)$$

$$d_{\text{mf}} = \frac{\int \mathbf{x}_n dV}{V} \quad (6)$$

Here, A_f denotes the interface area between fracture and matrix, m^2 ; k_{mf} is the harmonic mean of fracture and matrix permeability, m^2 ; d_{mf} represents the average distance from fractures to the matrix, m ; V is volume of the matrix unit, m^3 ; dV is the volume element of the matrix, m^3 ; $\mathbf{x}_n$ is distance from the dV within the matrix to the fracture, m .

Similarly, for NNC2, we have:

$$T_{\text{ff}} = \frac{T_{f1} T_{f2}}{T_{f1} + T_{f2}}, T_{f1} = \frac{k_{f1} w_{f1} L_{f1}}{d_{f1}}, T_{f2} = \frac{k_{f2} w_{f2} L_{f2}}{d_{f2}} \quad (7)$$

where T_{ff} represents the intersection transmissibility of fracture 1 and 2, m^3 ; k_{f1} , k_{f2} indicate fracture permeability of fracture segment 1 and fracture segment 2, m^2 , respectively; w_{f1} , w_{f2} are the aperture of fracture segment 1 and fracture segment 2, m ; L_{f1} , L_{f2} stand for the length of fracture segment 1 and fracture segment 2, m ; d_{f1} , d_{f2} are the average characteristic distance of fracture segment 1 and fracture segment 2, m .

For NNC3, the transmissibility can be defined as:

$$T_f = \frac{T_1 T_2}{T_1 + T_2}, T_1 = \frac{k_f A_c}{d_{\text{seg1}}/2}, T_2 = \frac{k_f A_c}{d_{\text{seg2}}/2} \quad (8)$$

where T_f is the transmissibility of adjacent fracture segments within a certain fracture segment, m^3 ; A_c is the cross-sectional area of the adjacent fracture segments, m^2 ; d_{seg1} , d_{seg2} denote the lengths of adjacent segments within a single fracture, m .

For fracturing fractures, it is necessary to introduce NNC4 to describe the fluid exchange between the wellbore and intersecting fractures. The wellbore-fracture flow rate is expressed as:

$$q_{\text{fs}} = \frac{T_{\text{fs}}}{\mu B} (p_{\text{wf}} - p_f) \quad (9)$$

$$T_{\text{fs}} = \frac{2\pi k_f w_f}{\ln(x_e/r_w) + S} \quad (10)$$

$$x_e = 0.14 \sqrt{h_f^2 + L_f^2} \quad (11)$$

where T_{fs} denotes the transmissibility between the well and the fractures, m^3 ; x_e is the equivalent radius, m ; p_{wf} is the bottomhole pressure, Pa ; r_w is the wellbore radius, m ; S is the skin factor; h_f and L_f are the height and length of fracture segment, m , respectively.

2.2. Discrete nonlinear system

In this study, the fluid is considered slightly compressible, and gravity is neglected. Therefore, the discrete equations can be derived using the finite volume method (FVM) and two-point flux approximation, as shown in Eq (12).

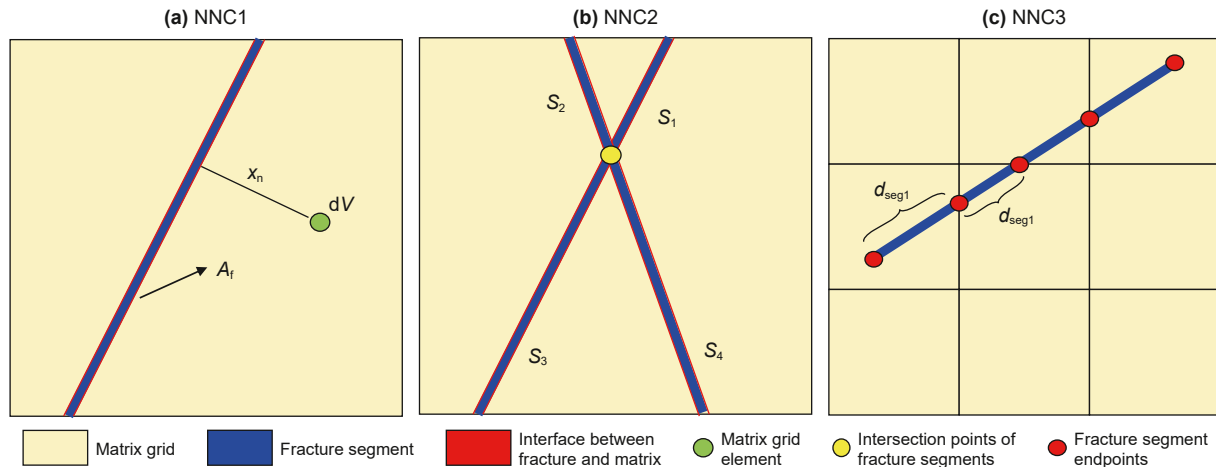


Fig. 1. Schematic diagram of three NNCs. (a) NNC between matrix and fracture, (b) NNC within intersecting fracture segments, (c) NNC between adjacent segments of the same fracture.

$$\begin{cases} \sum \frac{T_{mji}}{\mu} (p_{m,j}^{n+1} - p_{m,i}^{n+1}) + q_{ms,i}^{n+1} B - q_{mf,i}^{n+1} \\ = \frac{\phi_{m,i} C_t V_{m,i}}{\Delta t} (p_{m,i}^{n+1} - p_{m,i}^n) \\ \sum \frac{T_{fji}}{\mu} (p_{f,j}^{n+1} - p_{f,i}^{n+1}) + q_{fs,i}^{n+1} B - q_{fm,i}^{n+1} - q_{ff,i}^{n+1} \\ = \frac{\phi_{f,i} C_t V_{f,i}}{\Delta t} (p_{f,i}^{n+1} - p_{f,i}^n) \end{cases} \quad (12)$$

where T_{mji} and T_{fji} denote the transmissibility between the i -th and j -th matrix and fracture grids, m^3 , respectively; $p_{m,i}^{n+1}$ and $p_{f,i}^{n+1}$ represent pressure of the i -th matrix and fracture grid at $(n+1)$ -th time step, Pa, respectively; $\phi_{m,i}$ and $\phi_{f,i}$ are the porosity of the i -th matrix and fracture grid, respectively; $V_{m,i}$ and $V_{f,i}$ are the volume of the i -th matrix and fracture grid, m^3 , respectively; $q_{ms,i}^{n+1}$ and $q_{fs,i}^{n+1}$ represent source/sink terms for the i -th matrix and fracture grid, m^3/s , respectively; C_t is total compressibility of the rock-fluid system, Pa^{-1} .

Based on the EDFM discretization, the coupled matrix-fracture flow system can be written in a fully discrete form. The flux, source/sink, and accumulation terms are assembled into a nonlinear system, which is expressed as:

$$\begin{bmatrix} \mathbf{A}_{mm} & \mathbf{A}_{fm} \\ \mathbf{A}_{mf} & \mathbf{A}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{p}_m \\ \mathbf{p}_f \end{bmatrix} + \begin{bmatrix} \mathbf{Q}_{ms} \\ \mathbf{Q}_{fs} \end{bmatrix} = \begin{bmatrix} \mathbf{Acc}_m & \mathbf{0} \\ \mathbf{0} & \mathbf{Acc}_f \end{bmatrix} \begin{bmatrix} \Delta \mathbf{p}_m \\ \Delta \mathbf{p}_f \end{bmatrix} \quad (13)$$

where $\mathbf{A}_{mm}$ and $\mathbf{A}_{ff}$ represent intra-domain transmissibility matrices, while $\mathbf{A}_{mf}$ and $\mathbf{A}_{fm}$ describe matrix-fracture interactions; $\mathbf{Acc}_m$ and $\mathbf{Acc}_f$ are the accumulation coefficient matrices, $\Delta \mathbf{p}_m$ and $\Delta \mathbf{p}_f$ represent the pressure increments in the matrix and fracture cells, $\mathbf{Q}_{ms}$ and $\mathbf{Q}_{fs}$ denotes the perforation flow rate in matrix and fracture cells, respectively. Notably, $\mathbf{Q}_{ms}$ bears nonzero values for vertical wells only, whereas $\mathbf{Q}_f$ bears nonzero values solely at perforated hydraulic fractures.

Eq. (13) can be rewritten in residual form as:

$$F(x) = 0 \quad (14)$$

where F is the nonlinear residual function of the system, and $x = [p_m, p_f]^T$ is the unknown vector to be solved.

Eq. (14) is typically solved using the Newton–Raphson method. At the k -th iteration, a first-order Taylor expansion of the residual function yields:

$$J(x^k) \Delta x^k = -F(x^k) \quad (15)$$

where $J(x^k) = \partial F / \partial x$ is the Jacobian.

The solution is then updated as:

$$x^{k+1} = x^k + \Delta x = x^k + J(x^k)^{-1} (-F(x^k)) \quad (16)$$

Iteration continues until the convergence criterion is satisfied. Nevertheless, constructing and factorizing the Jacobian imposes substantial computational overhead for large-scale EDFM systems. To mitigate this bottleneck, we propose a neural-network-based approximation strategy for Newton updates in the subsequent section.

3. Methodology: PIARNN framework

This section presents the PIARNN framework for MFHW simulation. As shown in Fig. 2, the workflow is elaborated as

follows: First, fundamental field data are collected from in-situ geological archives and well test measurements, including reservoir geometry dimensions, matrix permeability, as well as fracture aperture, half-length and fracture conductivity. Using these parameters, the EDFM is implemented to discretize the coupled matrix-fracture system within a cartesian grid, whereby grid topological attributes and NNC transmissibility coefficients are calculated. Additional well control schedules and time-step settings are subsequently imported. The proposed PIARNN framework is then deployed for iterative computation to resolve transient reservoir pressure distribution and bottomhole production performance.

Compared against conventional PINNs for seepage equations, PIARNN features three innovations: (1) An adaptive factor is devised to substitute pressure input, mitigating progressive error accumulation throughout iterative computation. (2) A dual-branch network is constructed to separately characterize state variables of matrix and fracture subsystems, eliminating cross-interference between two disparate flow domains. (3) A refined incremental learning paradigm is incorporated to boost convergence robustness. Remarkably, PIARNN avoids Jacobian assembly and labeled datasets. All physical constraints are only imposed by the flow governing PDEs, so the model can reach the same numerical accuracy as Newton-based iterative solvers.

3.1. PIARNN architecture

In this work, we propose a paradigm: instead of directly approximating the solution p^{n+1} (as in conventional PINNs), we train a neural network to approximate the Newton update increment Δp^{n+1} , which serves as a surrogate for the iterative solver.

For each time step, the pressure increment is approximated as:

$$\Delta p^{n+1} \approx f_{\text{PIARNN}}(\lambda; \theta) \quad (17)$$

where $\lambda = [\lambda_m, \lambda_f]^T$ is the adaptive vector encoding the current system state. This vector consists of a set of randomly initialized parameters, which function similarly to the initial guess value in standard Newton iteration. During PIARNN training, λ is dynamically updated with all other trainable weights of the network. θ denotes the trainable network parameters. The unknown solution corresponding to time step $n+1$ is subsequently updated by the following formula:

$$p^{n+1} = p^n + f_{\text{PIARNN}}(\lambda; \theta) \quad (18)$$

The architecture of the proposed PIARNN is shown in Fig. 3. First, the 2-norm of the difference between λ and the center vector factor c_i is calculated, then scale the result with the adaptive scale factor s_i . In this work, two adaptive modules are adopted to separately learn the pressure increments of matrix and fracture grids.

$$d_m = \frac{\|w_1 \lambda_m - c_i\|_2}{s_i} \quad (19)$$

$$d_f = \frac{\|w_2 \lambda_f - c_i\|_2}{s_i} \quad (20)$$

where c_i is the center factor, s_i is the scale factor, w_i ($i = 1, 2, 3, 4$) are weight factors for feature capture.

Subsequently, the cosine-modulated Gaussian radial basis function (RBF) is adopted as feature activation, which ensures smooth propagation of extracted features:

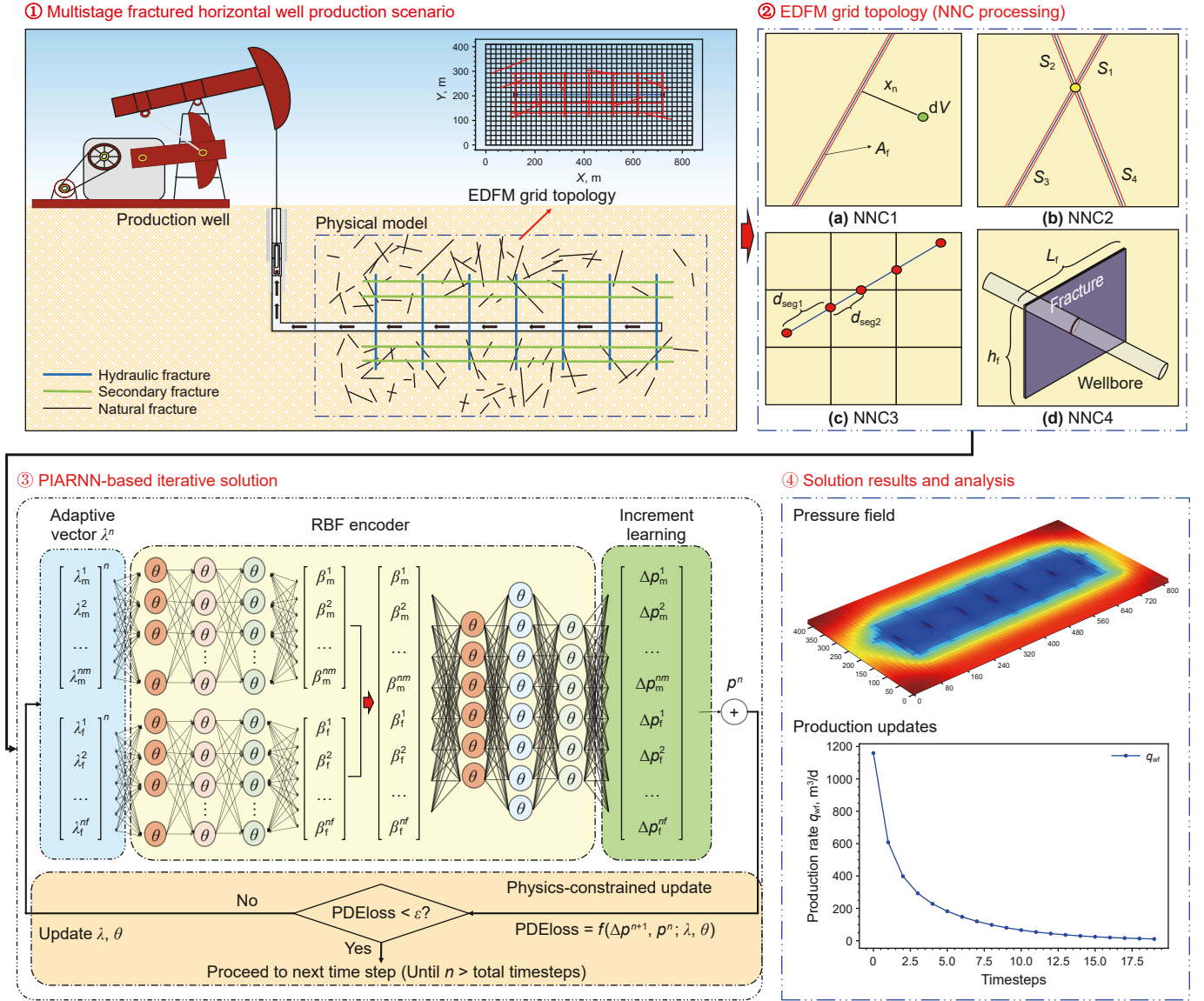


Fig. 2. Flowchart of the PIARNN solution framework.

$$\psi(\tau) = \cos(\omega_0 \tau) \exp(-\tau^2 / 2) \quad (21)$$

where ω_0 denotes the learnable frequency modulation parameter. This cosine-modulated Gaussian RBF retains the radial symmetry and local approximation property of standard RBFs, while introducing an oscillatory term to improve the network's performance in capturing the complex flow dynamics of fractures.

Further feature mapping is implemented via fully connected layers. The transformation formulas for the matrix and fracture branches are given below:

$$\begin{aligned} y_{i+1,m} &= W_{i,1} \cdot [w_3 \cdot \psi(d)] + b_{i,1} \\ y_{i+1,f} &= W_{i,2} \cdot [w_4 \cdot \psi(d)] + b_{i,2} \end{aligned} \quad (22)$$

After concatenating the feature outputs of the matrix and fracture branches, a final fully connected layer is applied for mapping and generate the network output:

$$y_{i+1} = W_{i,3} \cdot [y_{i+1,m}, y_{i+1,f}] + b_{i,3} \quad (23)$$

where W_{ij} ($j = 1, 2, 3$) and b_{ij} ($j = 1, 2, 3$) denote the weights and biases of the fully connection layer, respectively.

3.2. Physics-constrained training

The overall pressure-solving workflow of the proposed PIARNN is illustrated in Algorithm 1. Specifically, the network first performs adaptive feature extraction for the matrix-fracture systems using learnable adaptive factors. Next, the cosine-modulated Gaussian RBF strengthens its capacity to capture high-frequency pressure variations. Finally, fully connected layer realizes the nonlinear mapping of the pressure field. PIARNN removes the reliance on the current-step pressure input. It directly yields the high-fidelity pressure solution at the next time step through end-to-end learning of three groups of adaptive factor parameters,

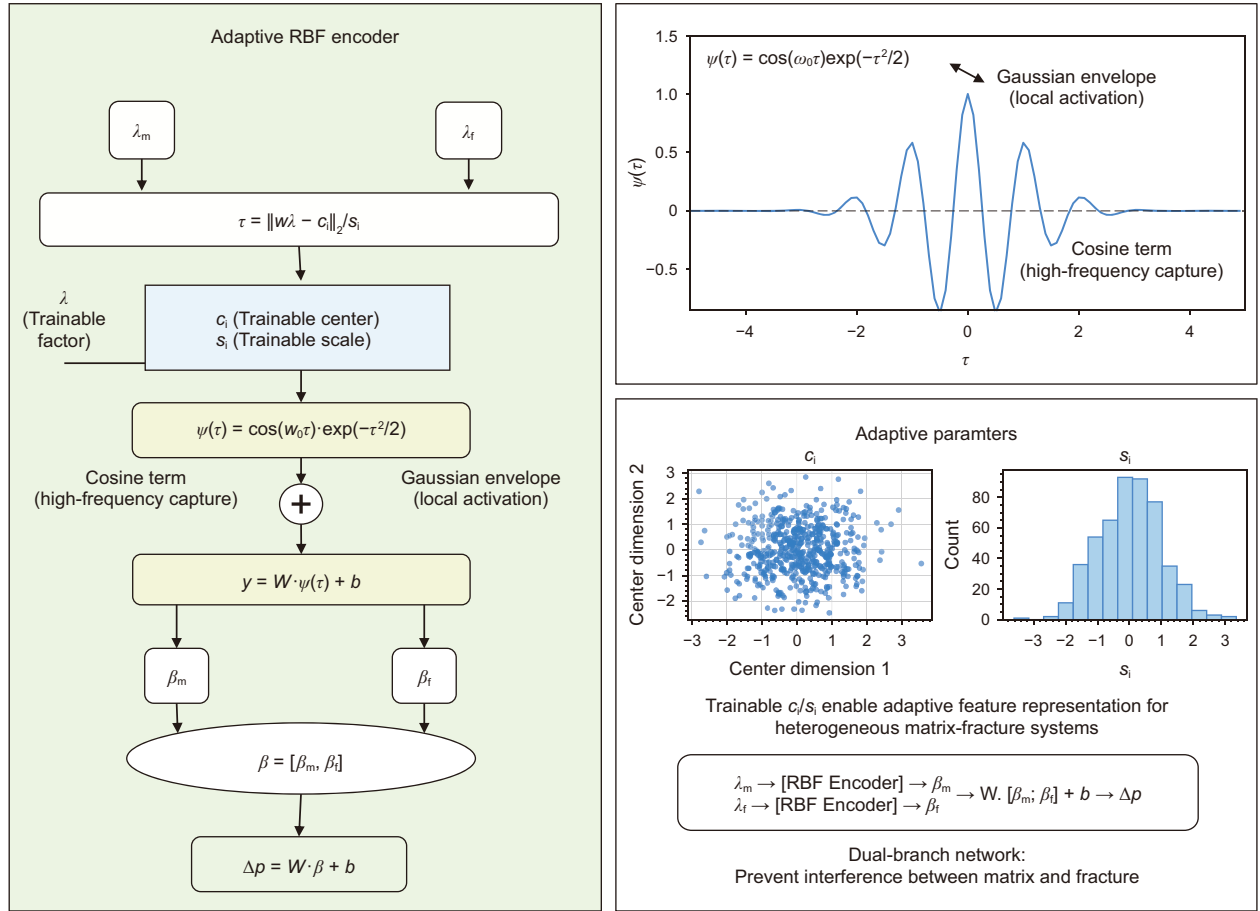


Fig. 3. Schematic detailed structure of PIARNN.

thereby eliminating error accumulation caused by sequential recursion.

To enforce physical consistency of model outputs, a loss function integrated with EDFM governing equations is constructed, as shown in Eq. (24). The loss term is incorporated into back-propagation process through Jacobian-free Newton iteration. As a result, the dependence on pressure-field inputs at the current time step is eliminated. The internal and external boundary conditions, including constant-pressure and constant-flux boundaries, are discretized directly in Eq. (25) using the FVM. Consequently, no additional boundary-condition loss terms are introduced.

$$\mathcal{L}_{\text{PDE}} = \frac{1}{N_t} \left\| f_{\text{PDE}}(\Delta p^{n+1}, p^n) \right\|_2^2 \quad (24)$$

$$f_{\text{PDE}}(\Delta p^{n+1}, p^n) = \begin{bmatrix} \mathbf{A}_{\text{mm}} & \mathbf{A}_{\text{fm}} \\ \mathbf{A}_{\text{mf}} & \mathbf{A}_{\text{ff}} \end{bmatrix} \underbrace{\begin{bmatrix} \mathbf{p}_{\text{m}} \\ \mathbf{p}_{\text{f}} \end{bmatrix}}_{p^n + \Delta p^{n+1}} + \begin{bmatrix} \mathbf{Q}_{\text{ms}} \\ \mathbf{Q}_{\text{fs}} \end{bmatrix} - \begin{bmatrix} \mathbf{Acc}_{\text{m}} & \mathbf{0} \\ \mathbf{0} & \mathbf{Acc}_{\text{f}} \end{bmatrix} \underbrace{\begin{bmatrix} \Delta \mathbf{p}_{\text{m}} \\ \Delta \mathbf{p}_{\text{f}} \end{bmatrix}}_{\Delta p^{n+1}} \quad (25)$$

Algorithm 1. Training procedure of PIARNN for EDFM reservoir simulation.

Input: Iterations N , time steps T , reservoir parameters, well and fracture parameters
Pre: Max iterations N_{max} , time steps T , reservoir/fracture/well parameters, EDFM grid
Output: Predicted matrix-fracture pressure field at target time step p^{n+1}
Preprocess: Assemble EDFM governing equations; Initialize PIARNN parameters, adaptive factors
Procedure $\text{PIARNN}(T, N_{\text{max}})$
for $t \leftarrow 1$ **to** T , **do**
 $\Delta p^{n+1} = f_{\text{PIARNN}}(\lambda^n, \theta)$ # λ : learnable adaptive factors; θ : network trainable parameters
 $\mathcal{L}_{\text{PDE}} \leftarrow \frac{1}{N_t} \|f_{\text{PDE}}(\Delta p^{n+1}, p^n)\|_2^2$ # Calculate EDFM physics-constrained loss
While $\mathcal{L}_{\text{PDE}} > \varepsilon$ **and** **epoch** $< N_{\text{max}}$ **do**
Update λ & θ via Adam optimizer ($\mathcal{L}_{\text{PDE}}$)
 $\Delta p^{n+1} \leftarrow f_{\text{PIARNN}}(\lambda, \theta)$ # Re-predict pressure increment
End while
 $p^{n+1} \leftarrow p^n + \Delta p^{n+1}$
 # Obtain final pressure field (no sequential dependency on p^n)
End for
Return p^{n+1}
End procedure

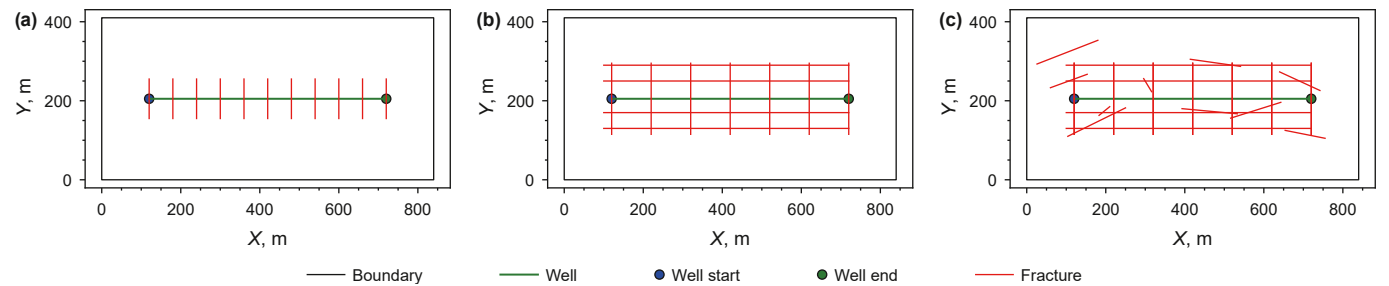


Fig. 4. Schematic diagram of three types of fracturing scenarios. (a) Case A: Simple hydraulic fracture; (b) Case B: Complex fracture network; (c) Case C: Complex fracture network connecting natural fractures.

4. Numerical experiments

According to the complexity of fracture systems, three typical scenarios (Fig. 4) are established, and the PIARNN is adopted to solve reservoir production dynamics. Basic parameters are listed in Table 1, case-specific parameters will be specified in corresponding examples. Case A represents a simple scenario with only primary fractures; Case B corresponds to a moderately complex scenario with fracture networks; Case C describes a strongly heterogeneous scenario where artificial fractures interconnect with natural fractures. For each case, the necessity and engineering representativeness are elaborated, and the simulation accuracy of PIARNN are evaluated systematically.

4.1. Case A: Simple fracturing of fractured horizontal well

During hydraulic fracturing in low-permeability reservoirs, long fractures extend bilaterally from the wellbore, providing high-conductivity flow pathways. Formation fluids migrate from the matrix into hydraulic fractures before converging toward the wellbore. For formations with sparse natural fractures and low rock brittleness, only symmetric bilateral hydraulic fractures propagate, without the development of complex fracture networks. Accordingly, Case A corresponds to this basic hydraulic fracturing scenario, and the PIARNN is used to simulate its production dynamic. Detailed fracture parameters are listed in Table 2.

The simulation contains 500 time steps with a 10-h interval, and pressure field results presented in Fig. 5. As production continues, pressure disturbances first propagate near the wellbore and fractures, then gradually expands to the whole drainage area and finally reaches the no-flow boundaries. The results prove that

Table 1
Default parameters for reservoir simulation.

Type	Parameter	Symbol	Unit	Value
Reservoir	Reservoir size	L_x, L_y	m	820×410
	Initial pressure	p_i	MPa	30
	Thickness	h	m	10
	Total compressibility	C_t	1/MPa	0.0535
Rock	Porosity	ϕ	/	0.2
	Permeability	K_m	mD	10
Fluid	Oil phase volume factor	B	m ³ /m ³	1.2
	Oil phase viscosity	μ	mPa·s	1.5
Wellbore	Wellbore radius	r_w	m	0.1
	Skin factor	S	/	0.5
	Well length	L	m	600
Well control	Surface flow rate	q_{sc}	m ³ /d	30
Schedule	Production time	$Time$	h	5000
	Time step	dt	h	10

Note: 1 MPa = 10⁶ Pa, 1 mD = 9.86 × 10⁻¹⁶ m², 1 mPa·s = 10⁻³ Pa·s.

Table 2
Fracture parameters for Case A.

Type	Parameter	Symbol	Unit	Value
Matrix	Grid number	$N_x \times N_y$	/	41 × 21
Primary fracture	Fracture number	$N_{f,pri}$	/	11
	Fracture permeability	$K_{f,pri}$	mD	5,000,000
	Fracture half-length	$X_{f,pri}$	m	50
	Fracture width	$w_{f,pri}$	m	0.001
	Fracture height	$h_{f,pri}$	m	10

Note: The subscript pri is the abbreviation of primary.

PIARNN can accurately capture the characteristic flow physics of fractured horizontal wells.

Model predictions are compared against reference solutions from the open-source reservoir simulator MRST, as illustrated in Fig. 6. The pressure distributions calculated by PIARNN and MRST show highly agreement, with a mean absolute error (MAE) of merely 5.08 × 10⁻⁵ MPa. The bottomhole pressure (BHP) profiles nearly fully overlap (Fig. 7). These results confirm that PIARNN achieves satisfactory accuracy and reliable performance in Case A.

4.2. Case B: Complex fracture network of fractured horizontal wells

For Case A, the PIARNN model exhibits favorable simulation performance. On this basis, we further extend the numerical investigation to Case B. In reservoirs with high rock brittleness, hydraulic fractures propagate laterally and vertically and trigger intensive rock fragmentation, which facilitates the generation of intricate fracture networks. Such fracture architectures significantly enlarge the drainage region, where fluids from secondary fractures converge toward primary hydraulic fractures.

To accurately capture the transient pressure evolution within the complex fracture network, the grid resolution is strategically refined. Relative to Case A, the number of primary fractures is reduced, while their lengths are extended. Specifically, four groups of secondary fractures are arranged orthogonally to the primary fractures, constructing a comprehensive fracture system. The corresponding fracture parameters are summarized in Table 3.

PIARNN is utilized for the simulation over 500 time steps, and the pressure field distributions at various production stages are presented in Fig. 8. It is observed that pressure propagation is not limited merely to the vicinity of primary hydraulic fractures, but spreads gradually from the hydraulic fractures into the entire complex fracture network. Secondary fractures closer to the primary fractures respond to pressure changes earlier. With sustained production, pressure disturbances ultimately occupy the whole stimulated reservoir volume.

Comparisons against reference numerical results from MRST are plotted in Fig. 9. The results demonstrate that PIARNN achieves reliable predictive performance under complex fracture network

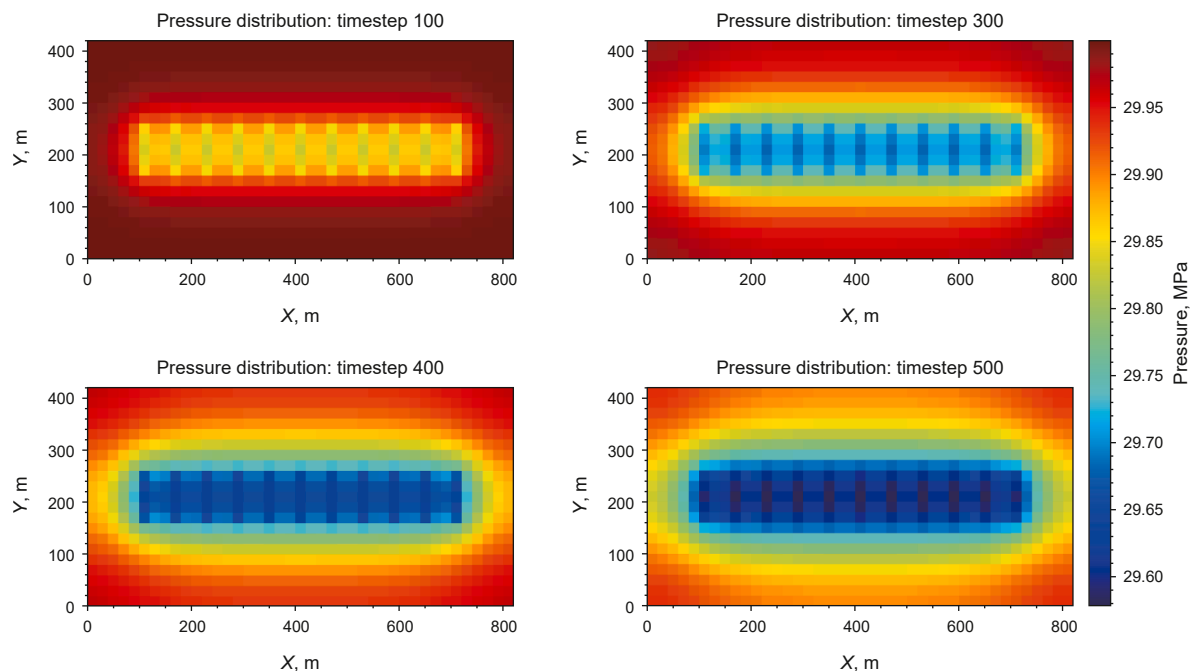


Fig. 5. Pressure distribution of different timesteps of Case A.

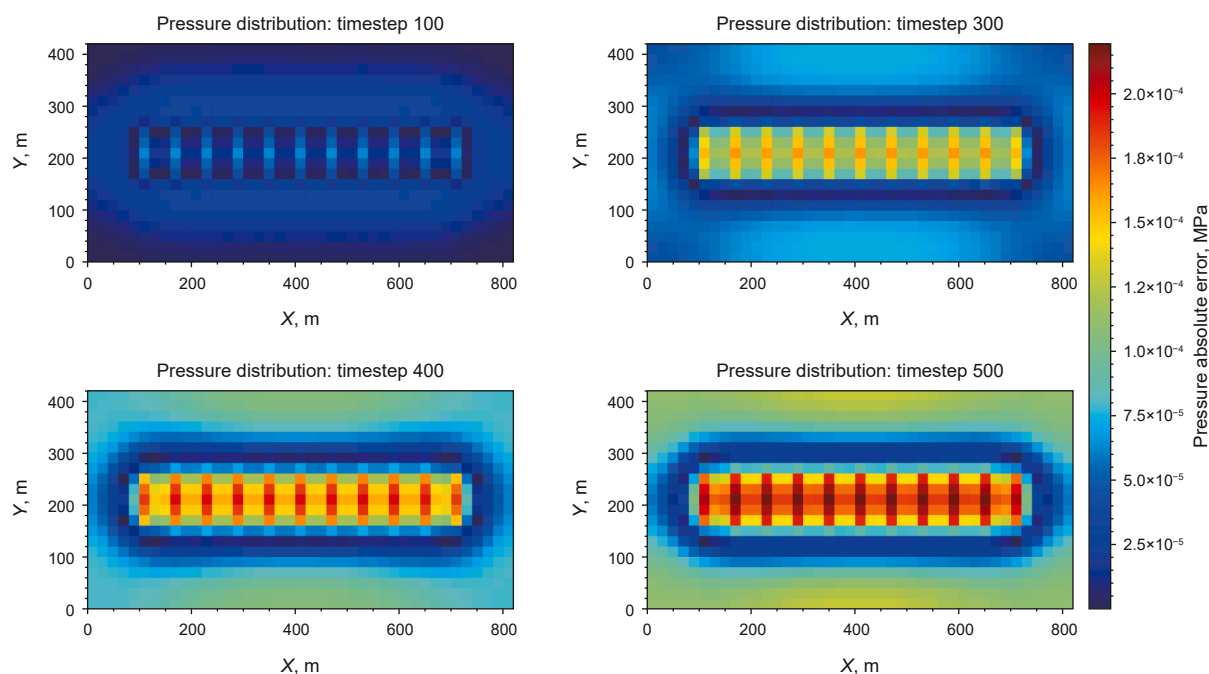


Fig. 6. Pressure absolute error of different timesteps of Case A.

conditions, with a MAE of 5.35×10^{-5} MPa. Its BHP profiles show nearly perfect agreement with MRST reference solutions (Fig. 10), validating the model's ability of to accurately capturing the flow dynamics within complex fracture networks.

4.3. Case C: Complex fracture network with natural fractures in fractured horizontal wells

The above two cases validate PIARNN's outstanding predictive performance for orthogonal fracture systems. For reservoirs with

abundant natural fractures, hydraulic fractures usually interact and intersect pre-existing natural fractures during fracturing operations, forming complex fracture networks coupled with natural fractures. Furthermore, natural fractures exhibit random spatial distribution. Their intersections with hydraulic fractures raise characterization difficulty and bring extra uncertainties to numerical simulations. To further test PIARNN's adaptability to complex fracture networks interconnected natural fractures, Case C is conducted. Relevant fracture parameters are listed in Table 4, and the coordinate of natural fractures is supplied in Appendix A.

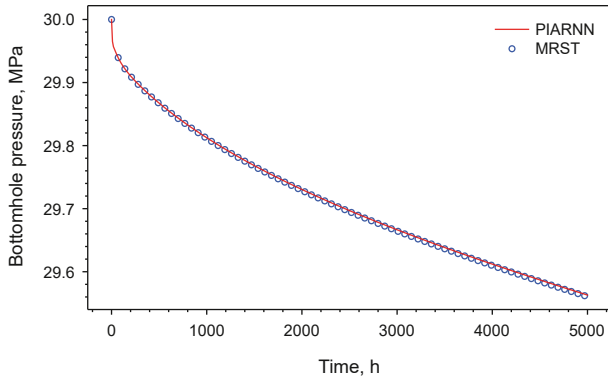


Fig. 7. Bottomhole pressure of Case A.

Table 3

Fracture parameters for Case B.

Type	Parameter	Symbol	Unit	Value
Matrix	Grid number	$N_x \times N_y$	/	61×41
Primary fracture	Fracture number	$N_{f,pri}$	/	7
	Fracture permeability	$K_{f,pri}$	mD	5,000,000
	Fracture half-length	$X_{f,pri}$	m	90
	Fracture width	$w_{f,pri}$	m	0.001
	Fracture height	$h_{f,pri}$	m	10
Secondary fracture	Secondary fracture number	$N_{f,sec}$	/	4
	Secondary fracture permeability	$K_{f,sec}$	mD	5,000,000
	Secondary fracture length	$L_{f,sec}$	m	620
	Secondary fracture width	$w_{f,sec}$	m	0.001
	Secondary fracture height	$h_{f,sec}$	m	10

Similarly, we carry out 500 time steps to investigate the adaptability of the proposed PIARNN, with the predicted pressure fields shown in Fig. 11. As development progresses, hydraulic fracture networks gradually link up surrounding natural fractures, which expands the effective seepage region.

Further comparisons with MRST benchmark solutions for fracture systems embedded with random natural fractures are provided in Figs. 12 and 13. PIARNN still delivers favorable predictive performance for intricate fracture systems embedded with natural fractures, with a MAE of 6.53×10^{-5} MPa. The outstanding matching degree of BHP curves further illustrates the strong generalization capacity of the proposed approach, supporting its applicability to MHFW with randomly distributed natural fractures.

4.4. Comprehensive analysis and variable-rate regime simulation

The BHP under identical production of three cases are presented in Fig. 14. Curve comparison shows that the consumed pressure drawdown declines from Case A to Case C, proving that fracturing stimulation effectiveness improves stepwise.

In this work, stimulation performance is embodied by primary fracture half-lengths, formation of intricate fracture networks, and connectivity with natural fractures. Notably, although Case A contains more primary fractures than the other two cases, its reservoir stimulation effect remains relatively weak. Cases B and C feature longer fracture half-lengths and form complex coupled fracture networks with superior natural fracture connectivity, thus delivering better stimulation outcomes.

Meanwhile, the PIARNN obtains high-accuracy predictions for the three cases with different fracture complexity. The fitted pressure evolution aligns with seepage principles, verifying that the model exhibits excellent numerical stability and generalization capability.

Stable production constraints are difficult to maintain during actual field development, and well control conditions usually evolve continuously throughout the development cycle. To investigate the adaptability of the PIARNN to production rate disturbances, a variable-rate simulation is carried out based on the fracture parameters of Case C. A variable-rate schedule is established to examine the adaptive performance of PIARNN, as shown in Fig. 15. Unlike the constant well-control scenarios in the

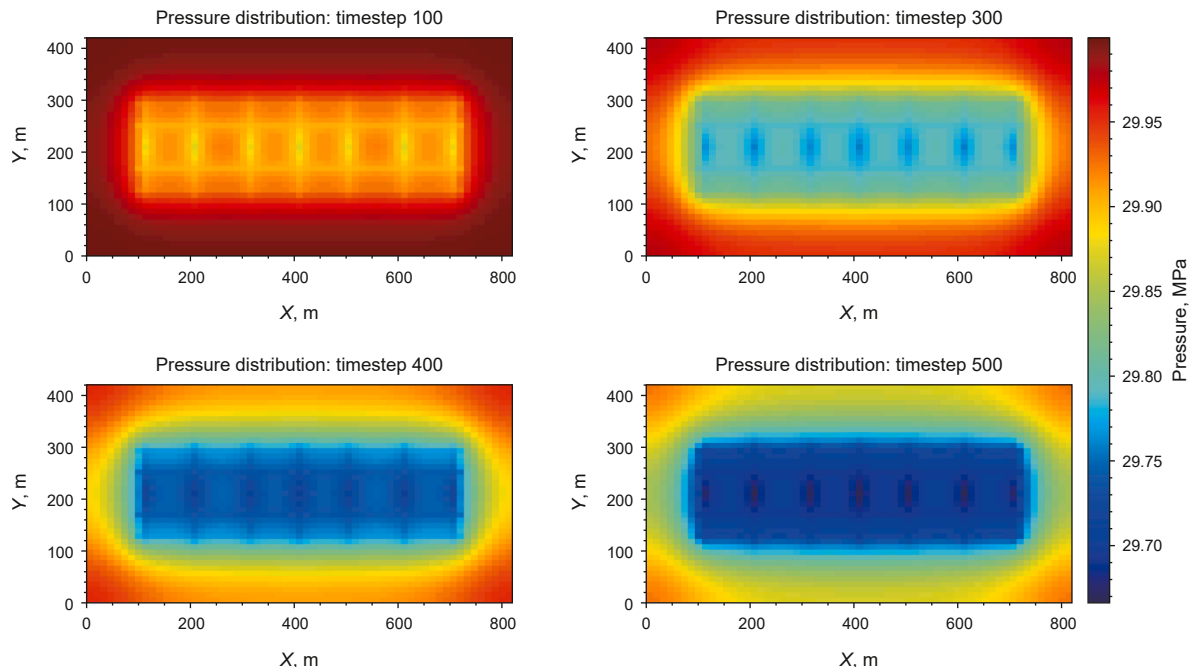


Fig. 8. Pressure distribution of different timesteps of Case B.

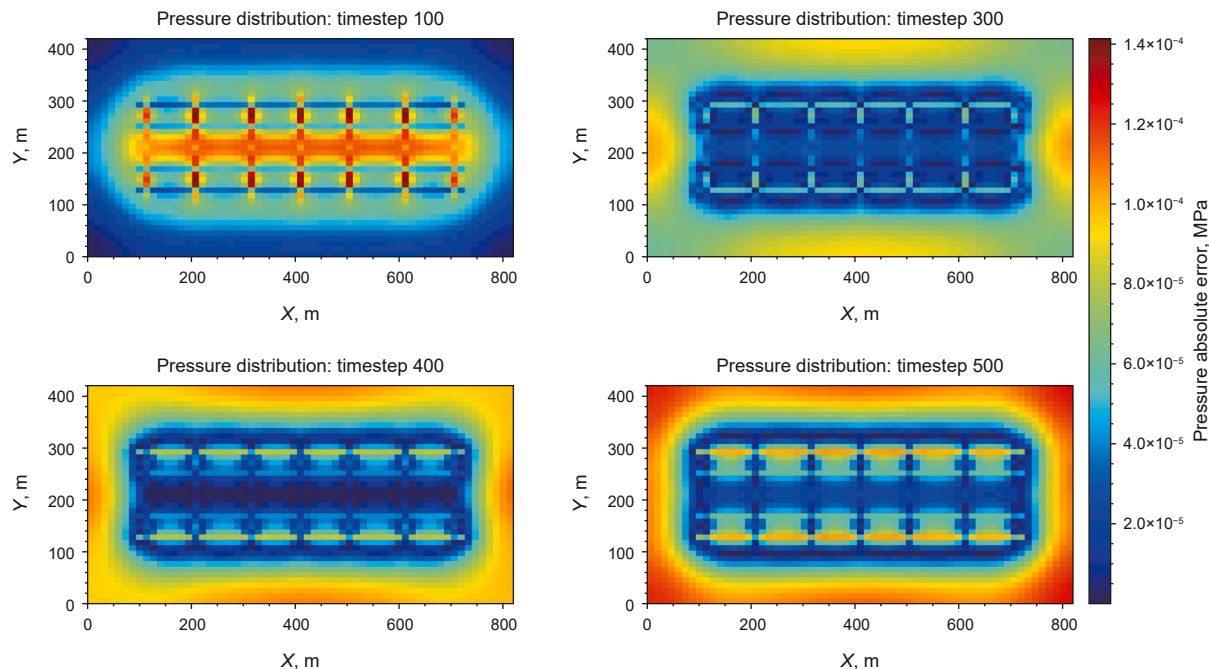


Fig. 9. Pressure error of different timesteps of Case B.

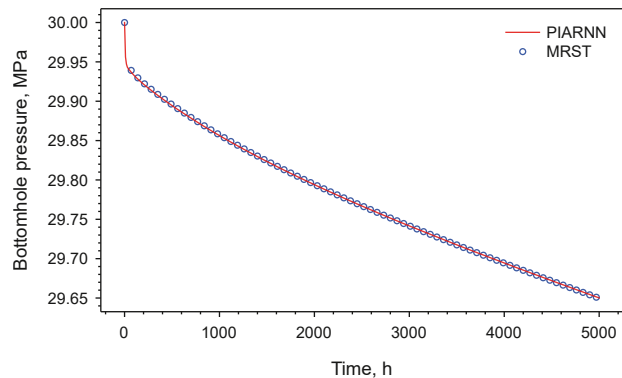


Fig. 10. Bottomhole pressure of Case B.

Table 4				
Fracture parameters for Case C.				
Type	Parameter	Symbol	Unit	Value
Matrix	Grid number	$N_x \times N_y$	/	61×41
	Fracture number	$N_{f,pri}$	/	7
Primary fracture	Fracture permeability	$K_{f,pri}$	mD	5,000,000
	Fracture half-length	$X_{f,pri}$	m	90
	Fracture width	$w_{f,pri}$	m	0.001
	Fracture height	$h_{f,pri}$	m	10
	Secondary fracture number	$N_{f,sec}$	/	4
Secondary fracture	Secondary fracture permeability	$K_{f,sec}$	mD	5,000,000
	Secondary fracture length	$L_{f,sec}$	m	620
	Secondary fracture width	$w_{f,sec}$	m	0.001
	Secondary fracture height	$h_{f,sec}$	m	10
	Natural fracture number	$N_{f,nat}$	/	10
Natural fracture	Natural fracture permeability	$K_{f,nat}$	mD	5,000,000
	Natural fracture width	$w_{f,nat}$	m	0.001
	Natural fracture height	$h_{f,nat}$	m	10

previous cases, the production rate undergoes 50 abrupt fluctuations across 2000 time steps.

The BHP results are displayed in Fig. 16. It can be observed that the predictions from PIARNN match the MRST reference solutions with excellent consistency. This demonstrates that PIARNN retains favorable numerical stability even under abrupt production rate fluctuations.

5. Discussion and performance analysis

The proposed PIARNN employs a dual-branch architecture to learn the pressure increments of the matrix and fracture systems, respectively. Notably, it does not rely on the pressure field from the previous time step during training; instead, it updates the pressure field at the next step only by optimizing three types of adaptive parameters. Benefiting from these key designs, PIARNN achieves satisfactory adaptability in all three scenarios.

To further quantitatively evaluate the performance of PIARNN, this section first analyzes its prediction accuracy and computational efficiency under different grid sizes. Subsequently, a comprehensive comparison between PIARNN and conventional neural networks is conducted. In addition, ablation studies are performed to reveal the contributions of core components and further validate the superiority of the proposed framework.

5.1. Accuracy and efficiency analysis

We compare the prediction error and solution time of the proposed PIARNN across three reservoir simulation scenarios with increasing fracture complexity, with the results presented in Table 5.

First, in terms of prediction accuracy, the pressure MAE of PIARNN remains at the order of 10^{-5} MPa for all three cases, which is sufficiently low to meet engineering accuracy requirements for reservoir numerical simulation. Notably, even for Case C with the most complex natural fracture network and the largest grid scale

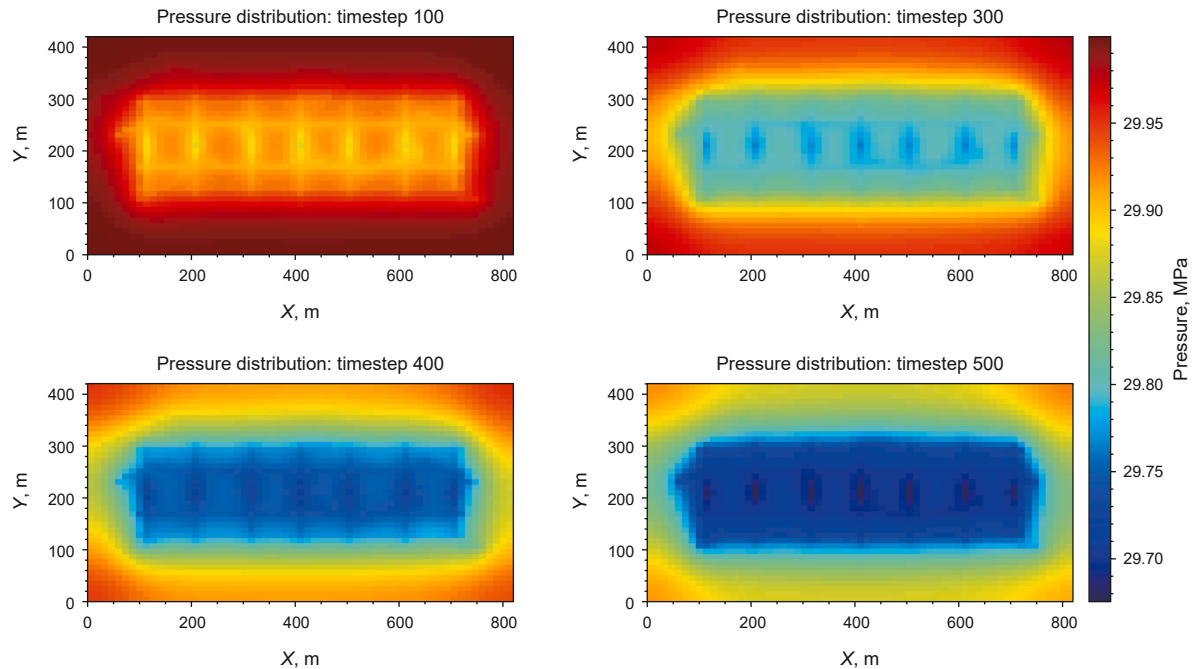


Fig. 11. Pressure distribution of different timesteps of Case C.

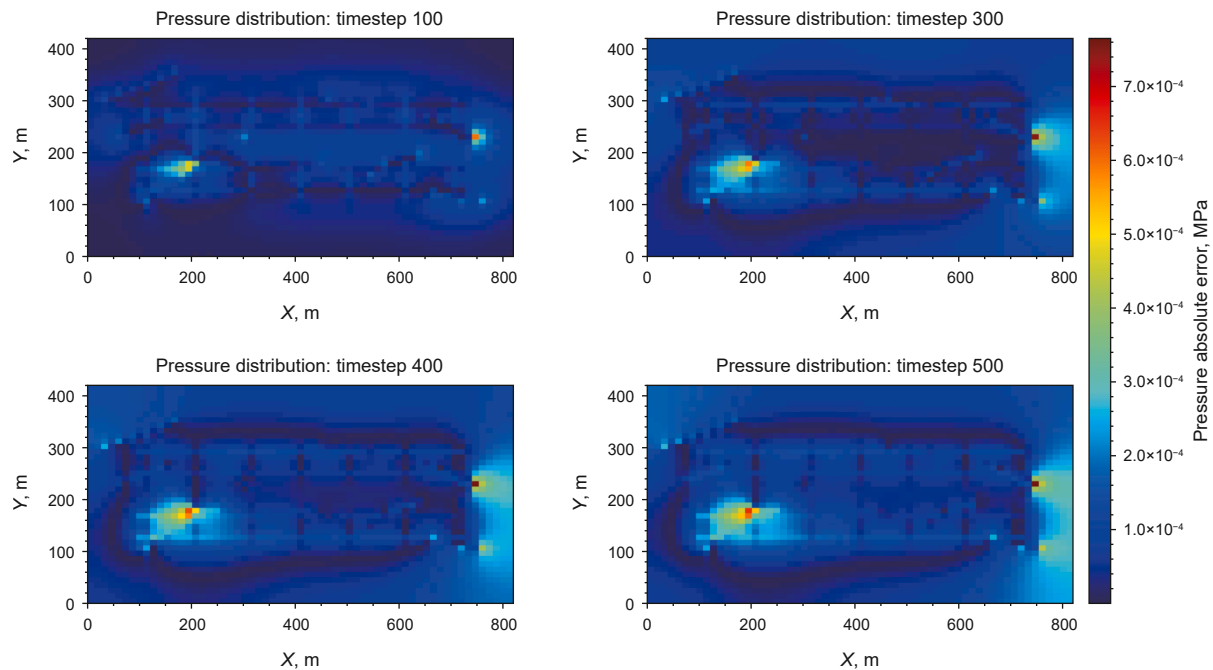


Fig. 12. Pressure absolute error of different timesteps of Case C.

(2945 grids), the MAE only increases slightly to 6.53×10^{-5} MPa, demonstrating that PIARNN maintains excellent stability and adaptability in complex fracture systems.

Second, regarding computational efficiency, the solution time increases gradually with grid scale and fracture complexity: from 242.21 s (Case A, simple fractures) to 310.94 s (Case B, complex fracture network), and 325.56 s (Case C, complex fracture network with natural fractures). Importantly, although the grid counts of Cases B and C are nearly three times that of Case A, the solution time does not rise significantly, and the

overall prediction accuracy remains stable. This indicates that PIARNN possesses strong computational scalability and achieves high accuracy in MFHW simulations for all three scenarios.

5.2. Comparative analysis of different networks

To demonstrate the superiority of PIARNN over PINN and PICNN, a comparative analysis is conducted under identical setups. The input and output details of comparative networks are provided in [Appendix A](#).

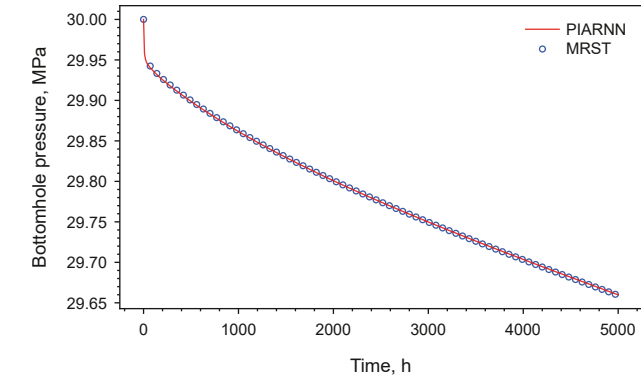


Fig. 13. Bottomhole pressure of Case C.

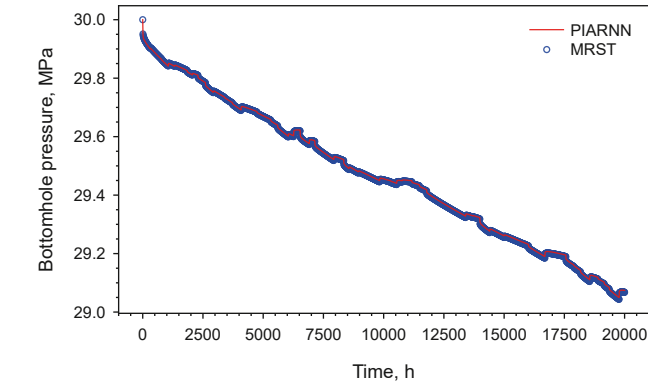


Fig. 16. Bottomhole pressure under variable production rates.

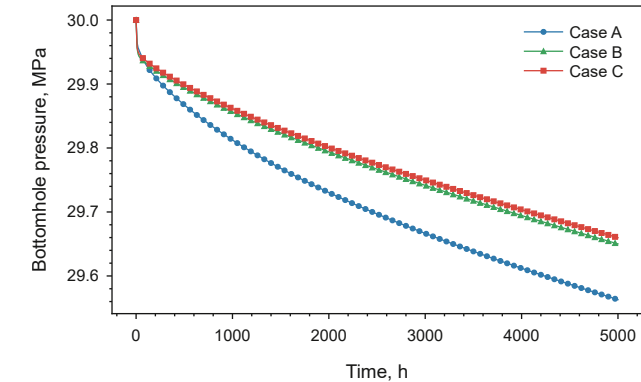


Fig. 14. Bottomhole pressure of three cases.

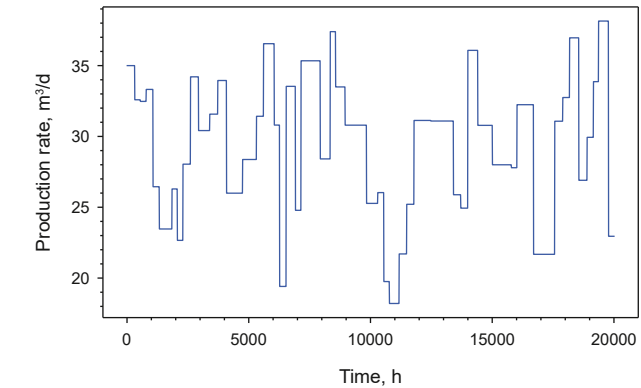


Fig. 15. Production rate at different times.

The first step of Case C is selected for result analysis for two key reasons. First, the pressure field at the initial time step is free from multi-step iterative evolution with minimal fluctuations, which enables accurate observation of subtle discrepancies. Second, as a complex natural fracture network scenario, Case C is a stringent test for network adaptability and robustness, and its first-time-step results are sufficient to distinguish model performance effectively.

As shown in Fig. 17, PIARNN solutions are in excellent agreement with the MRST benchmark. In contrast, the pressures solved by the PICNN and PINN both exceed 30 MPa, which is higher than the initial formation pressure and shows obvious physical

Table 5
Performance comparison of PIARNN and Newton–Raphson method for different cases.

Case	Total number of grids	Pressure MAE, MPa	Solution time, s
A	916	5.08×10^{-5}	242.21
B	2822	5.35×10^{-5}	310.94
C	2945	6.53×10^{-5}	325.56

inconsistencies. This demonstrates that the PIARNN has superior adaptability for complex MFHW simulation problems. Its dual-branch design for matrix and fracture system solving, coupled with adaptive incremental iteration, effectively guarantees the physical authenticity and fine-grained fitting capability of the solution results.

5.3. Ablation experiment

To investigate the core advantages of the proposed PIARNN, systematic ablation experiments are conducted. The full PIARNN is compared with three variant structures: the model without the adaptive factor, the model without the dual-branch architecture, and the model without incremental learning. The training MSE curves are presented in Fig. 18. It is observed that PIARNN consistently outperforms all the above variants by a considerable margin. Specifically, the model without incremental learning yields the largest training error, and the model without the dual-branch structure exhibits a comparable error level; both suffer from severe oscillations. The model without the adaptive factor performs slightly better but still shows unstable fluctuations during convergence. These results demonstrate that the adaptive factor effectively stabilizes the training process and ensures steady convergence, while the dual-branch structure and incremental learning strategy significantly improve the accuracy of EDFM-based MFHW simulation.

Furthermore, the BHP curves of the four configurations for Case C are presented in Fig. 19. It is evident that the results of PIARNN agree best with the reference solution from MRST, whereas the model without incremental learning performs the worst, followed by the one without the dual-branch structure, and then the one without the adaptive factor.

These results verify that the proposed PIARNN enables stable and accurate simulation for EDFM fractured models, which benefits from its three key designs:

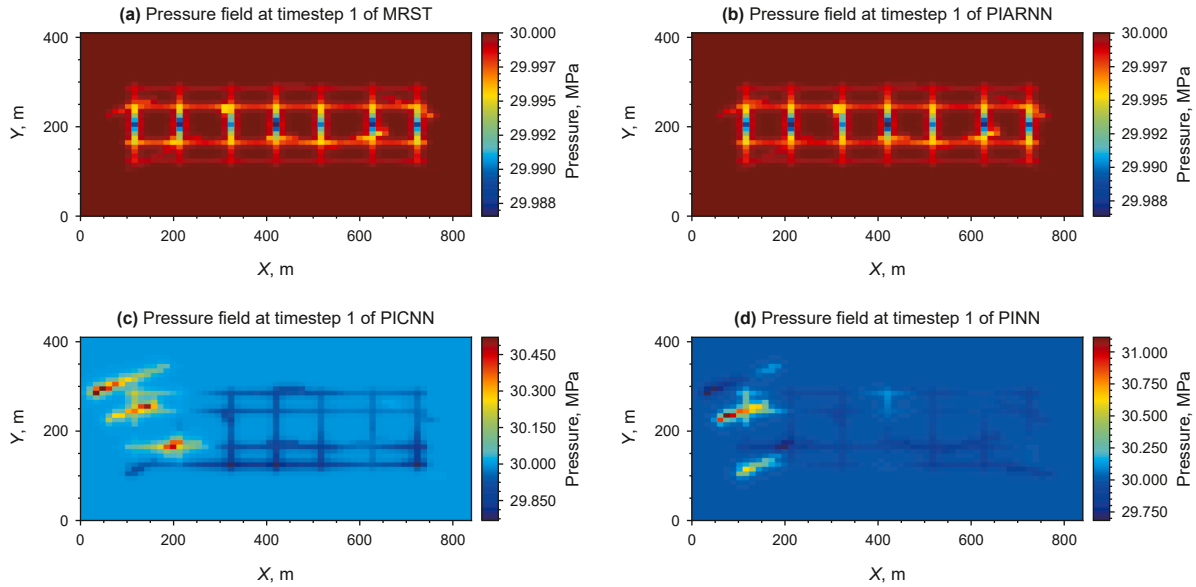


Fig. 17. Pressure fields at the first time step of Case C solved by different models.

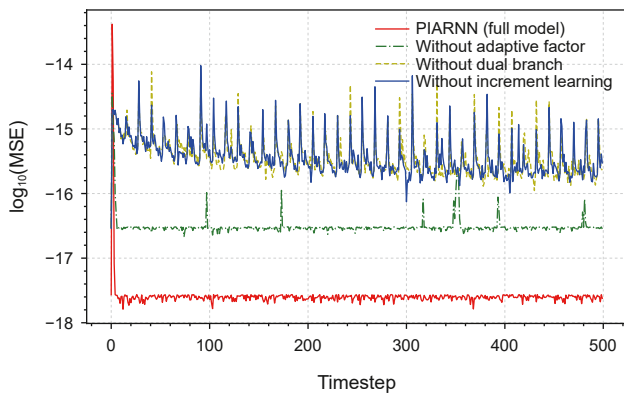


Fig. 18. Comparison of training MSE under different ablation experiments.

- (1) Adaptive factor: By eliminating dependence on the pressure value from the previous time step, it stabilizes training and reduces error accumulation.

- (2) Dual-branch architecture: It separately models matrix and fracture flow dynamics, mitigating strong mutual interference between matrix pressure and fracture pressure.
- (3) Incremental learning strategy: By learning pressure increments with a smaller numerical magnitude, the model converges more easily and captures subtle pressure variations more precisely.

5.4. Model limitations

From Table 5, the computational time of PIARNN reaches hundreds of seconds across the three test cases, indicating that this method cannot fully replace the traditional Newton method for engineering applications at the current stage. Despite this limitation, the core contribution of this study is to propose an innovative constraint strategy for surrogate model training, which effectively lessens the model's reliance on labeled simulation datasets.

Unlike conventional Newton solvers, the proposed method completely avoids the costly assembly and inversion of Jacobian. It leverages the nonlinear mapping capability of neural networks to

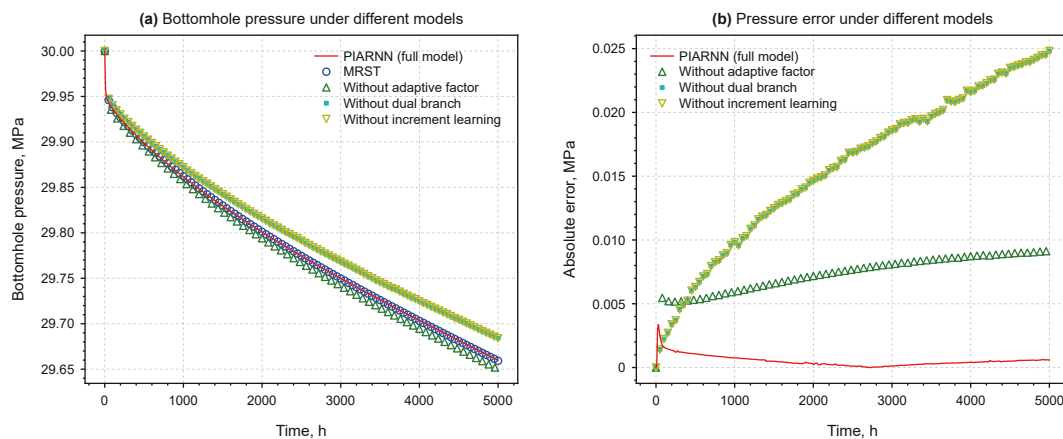


Fig. 19. Comparison of BHP prediction results for four model configurations (Case C).

characterize complex pressure transient behaviors within the coupled matrix-fracture system. With the continuous improvement of GPU computing performance, Jacobian-free solution frameworks are expected to become an emerging paradigm for future reservoir simulations.

Furthermore, the grid size of our test cases is much smaller than the million-cell meshes commonly used in field-scale numerical simulations. Extending the current model to large-scale grids remains a critical challenge for subsequent surrogate model optimization. Even so, the proposed scheme provides a feasible alternative to Newton solvers for solving high-dimensional nonlinear systems.

6. Conclusions

- (1) This study proposes a physics-informed adaptive radial basis neural network (PIARNN) for multistage fractured horizontal well simulation based on the Embedded Discrete Fracture Model (EDFM). The PIARNN introduces adaptive factors as inputs and adopts a dual-branch architecture to separately model pressures in matrix and fracture grids. Radial basis function layers are used for feature mapping, while fully connected layers predict pressure increments at the next time step. By embedding the discretized EDFM equations, the model avoids explicit Jacobian construction and provides an efficient alternative to Newton–Raphson methods.
- (2) The performance of PIARNN is validated on three fractured reservoir cases with increasing complexity: simple fracture, complex fracture network, and complex fracture network coupled with natural fractures. The predicted pressure fields show strong agreement with results from MRST, with MAEs on the order of 10^{-5} MPa. The bottomhole pressure curves are also well reproduced. In addition, the computational cost shows no significant increase with grid size, indicating good scalability. For variable-rate production problems, PIARNN also exhibits strong adaptability.
- (3) PIARNN achieves stable and accurate simulations for both simple and complex fracture systems. It performs particularly well in reservoirs with discrete natural fractures. Compared with PINN and PICNN, PIARNN shows the best agreement with MRST reference solutions. In contrast, PINN and PICNN exhibit reduced accuracy when handling complex fracture networks.
- (4) Ablation studies are conducted to examine the key components of PIARNN. Three simplified models are tested: without adaptive factors, without the dual-branch structure, and without incremental learning. The results show that adaptive factors improve stability and suppress oscillations, incremental learning enhances prediction accuracy, and the dual-branch structure reduces interference between matrix and fracture pressures. These components together ensure high accuracy and robustness.
- (5) The current PIARNN framework is limited to two-dimensional single-phase flow simulations. Future work will extend the model to three-dimensional and multiphase flow problem, and further apply it to history matching of reservoirs. Additionally, this model will be embedded into a surrogate modeling system to enhance computational efficiency for numerical simulation.

CRedit authorship contribution statement

Hai-Feng Zhu: Writing – review & editing, Writing – original draft, Conceptualization. **Zhi-Ming Chen:** Visualization,

Supervision, Funding acquisition. **Chen-Xin Huang:** Visualization, Validation. **Wei Yu:** Software, Methodology.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Nomenclatures

ρ	Fluid density, kg/m ³
q_{ms}	Source-sink term of matrix system, m ³ ·s ⁻¹
∇	Gradient operator
ϕ_m	Porosity
t	Time, s
$\mathbf{v}_m$	Flow velocity of fluid in the matrix system, m·s ⁻¹
q_{mf}	Cross-flow rate from the matrix system to the fracture system, m ³ ·s ⁻¹
V_m	Volume of matrix grid, m ³
ϕ_f	Fracture porosity
$\mathbf{v}_f$	Fluid flow velocity in the fracture system, m·s ⁻¹
q_{fs}	Source-sink term of fracture system, m ³ ·s ⁻¹
q_{ff}	Flow between intersecting fractures, m ³ ·s ⁻¹
V_f	Volume of fracture grid, m ³
B	Fluid formation volume factor, m ³ /m ³
ρ_{sc}	Fluid density under standard surface conditions, kg/m ³
K_α	Permeability, m ²
μ	Fluid viscosity, Pa·s
p_α	Pressure, Pa
g	Gravitational acceleration, m/s ²
T_{nnc}	NNCs transmissibility, m ³
p_j, p_i	Pressures of NNC grids and the local grid, Pa
A_f	Interface area between fracture and matrix, m ²
k_{mf}	Harmonic mean of fracture and matrix permeability, m ²
d_{mf}	Average distance from fractures to the matrix, m
V	Volume of the matrix unit, m ³
dV	Volume element of the matrix, m ³
$\mathbf{x}_n$	Distance from the dV within the matrix to the fracture, m
T_{ff}	Intersection transmissibility of fracture 1 and 2, m ³
k_f	Fracture permeability, m ²
k_{f1}, k_{f2}	Fracture permeability of fracture segment 1 and fracture segment 2, m ²
w_{f1}, w_{f2}	Aperture of fracture segment 1 and fracture segment 2, m
L_{f1}, L_{f2}	Length of fracture segment 1 and fracture segment 2, m
d_{f1}, d_{f2}	Average characteristic distance of fracture segment 1 and fracture segment 2, m
T_f	Transmissibility of adjacent fracture segments within a certain fracture segment, m ³
A_c	Cross-sectional area of the adjacent fracture segments, m ²
d_{seg1}, d_{seg2}	Length of adjacent segments within a single fracture, m
T_{fs}	Transmissibility between the well and the fractures, m ³
x_e	Equivalent radius, m
p_{wf}	Bottomhole pressure, Pa

r_w	Wellbore radius, m
S	Skin factor
h_f, L_f	Height and length of fracture segment, m
$T_{m,ji}, T_{f,ji}$	Transmissibility coefficients between the i -th and j -th matrix and fracture grids, m^3
$p_{m,i}^{n+1}, p_{f,i}^{n+1}$	Pressure of the i -th matrix and fracture grid at $(n+1)$ -th time step, Pa
$\phi_{m,i}, \phi_{f,i}$	Porosity of the i -th matrix and fracture grid, respectively
$V_{m,i}, V_{f,i}$	Volume of the i -th matrix and fracture grid, m^3
$q_{ms,i}^{n+1}, q_{fs,i}^{n+1}$	Source/sink terms for the i -th matrix and fracture grid, m^3/s
C_t	Total compressibility of the rock-fluid system, Pa^{-1}
A_{mm}, A_{ff}	Intra-domain transmissibility matrices
A_{mf}, A_{fm}	Describe matrix-fracture interactions
Acc_m, Acc_f	Accumulation coefficient matrices
$\Delta p_m, \Delta p_f$	Pressure increments in the matrix and fracture cells
Q_{ms}, Q_{fs}	Perforation flow rate in matrix and fracture cells
F	Nonlinear residual function of the system
$\lambda = [\lambda_m, \lambda_f]^T$	Adaptive vector encoding the current system state
θ	Trainable network parameters
c_i	Center factor
s_i	Scale factor
$w_i (i = 1, 2, 3, 4)$	Weight factors for feature capture
$W_{ij} (j = 1, 2, 3), b_i (i = 1, 2, 3)$	Weights and biases of the fully connection layer

Appendix A

Table A-1 details the complete network architectures adopted in this study. Note that comparative experiments between PICNN and PINN were not conducted for Case A and Case B. Additional fracture coordinate data for the stochastic fracture model in Case C are provided in Table A-2.

Table A-1
Neural network structures for different cases.

Model	Case	Input	Input layer	Hidden layer	Output
PIARNN	A	Adaptive factor 1 (1, 861) Adaptive factor 2 (1, 55)	ARNN 1 (861, 500) ARNN 2 (55, 500)	FC (500, 916)	Pressure increment (1, 916)
	B	Adaptive factor 1 (1, 2501) Adaptive factor 2 (1, 321)	ARNN 1 (2501, 500) ARNN 2 (321, 500)	FC (500, 2822)	Pressure increment (1, 2822)
	C	Adaptive factor 1 (1, 2501) Adaptive factor 2 (1, 444)	ARNN 1 (2501, 500) ARNN 2 (444, 500)	FC (500, 2945)	Pressure increment (1, 2945)
PICNN	A	Pressure at previous time step (1, 1, 916)	CNN1d (1, 25)	CNN1d (25, 50) FC (950, 916)	Pressure (1, 916)
	B	Pressure at previous time step (1, 1, 2822)	CNN1d (1, 25)	CNN1d (25, 50) FC (2950, 2822)	Pressure (1, 2822)
	C	Pressure at previous time step (1, 1, 2945)	CNN1d (1, 25)	CNN1d (25, 50) FC (3050, 2945)	Pressure (1, 2945)
PINN	A	Pressure at previous time step (1, 916)	FC (1, 916)	FC (916, 916) FC (916, 916) FC (916, 916) FC (916, 916)	Pressure (1, 916)
	B	Pressure at previous time step (1, 2822)	FC (1, 2822)	FC (2822, 2822) FC (2822, 2822) FC (2822, 2822) FC (2822, 2822)	Pressure (1, 2822)
	C	Pressure at previous time step (1, 2945)	FC (1, 2945)	FC (2945, 2945) FC (2945, 2945) FC (2945, 2945) FC (2945, 2945)	Pressure (1, 2945)

Table A-2
Coordinate information of fractures for Case C.

Fracture No.	X1, m	Y1, m	X2, m	Y2, m
1	26	293	180	353
2	60	233	153	267
3	640	273	742	226
4	393	180	533	167
5	183	163	210	185
6	516	156	643	196
7	296	256	316	223
8	104	110	250	182
9	654	125	755	105
10	414	305	541	287

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